

# **Intermediate Microeconomics (ECO 302)**

Lecture Notes

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# Chapter 1: The Market

## 1.1 Constructing a Model

### Key Idea

A **model** is a deliberately simplified representation of reality. The power of a model comes from stripping away irrelevant detail to focus on the mechanism of interest—in this chapter, how the *apartment market* determines price and allocation.

We study a medium-sized college town with two rings of apartments:

- *Inner ring* (near campus): scarce and highly valued.
- *Outer ring*: plentiful at a roughly fixed alternative price; serves as the “outside option.”

We focus on inner-ring apartments and ask:

1. What determines the **price** of these apartments?
2. Who gets them (the **allocation**)?

### Two principles guiding the analysis

- **Optimization**: Agents (consumers, landlords) choose the best available option given constraints.
- **Equilibrium**: Prices adjust so that choices are mutually compatible; market clears where quantity demanded equals quantity supplied.

## 1.2 Optimization and Equilibrium

### Key Idea

Price is a signal. Given a price, consumers optimally decide whether to rent an inner-ring apartment or take the outer-ring alternative; landlords optimally decide whether (and how much) to supply. *Equilibrium* price  $p^*$  equates these decisions.

## 1.3 The Demand Curve

**Demand** shows, for each price  $p$ , how many inner-ring apartments are desired if every renter takes the better of:

- inner-ring apartment at price  $p$ , vs.
- outside option (outer ring) at its fixed cost/quality.

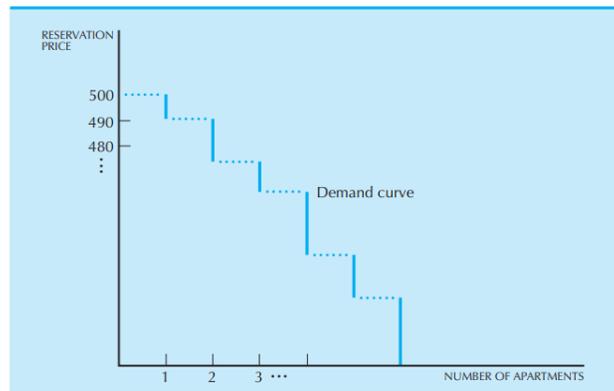


Figure 1.1: A step/declining *demand curve*: reservation prices across potential renters; height reflects willingness to pay.

**Reservation price** is the highest price at which a consumer is indifferent between an inner-ring unit and the outside option. Ordering renters by reservation price generates a downward-sloping market demand curve  $D(p)$ .

### Example

If the outside option costs \$480 and some students value proximity at \$20, \$30, \$50 more, then their reservation prices are \$500, \$510, \$530, etc. At  $p = \$500$ , only those with reservation  $\geq 500$  rent inside.

## 1.4 The Supply Curve

In the very short run, the number of inner-ring apartments is fixed at  $S$  (buildings can't instantly change). Hence:

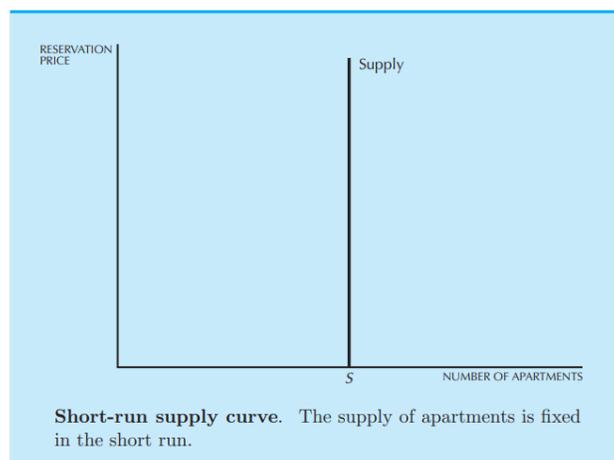


Figure 1.2: Short-run *vertical supply*: quantity is fixed at  $S$ .

**Short run:** supply is perfectly inelastic (vertical at  $S$ ). **Long run:** developers can add or remove units; supply can respond to price (we revisit later).

## 1.5 Market Equilibrium

### Key Idea

**Equilibrium price**  $p^*$  solves  $D(p^*) = S$ . At  $p^*$ , every unit supplied is taken by someone whose reservation value  $\geq p^*$ ; no one who values it less pays  $p^*$ .

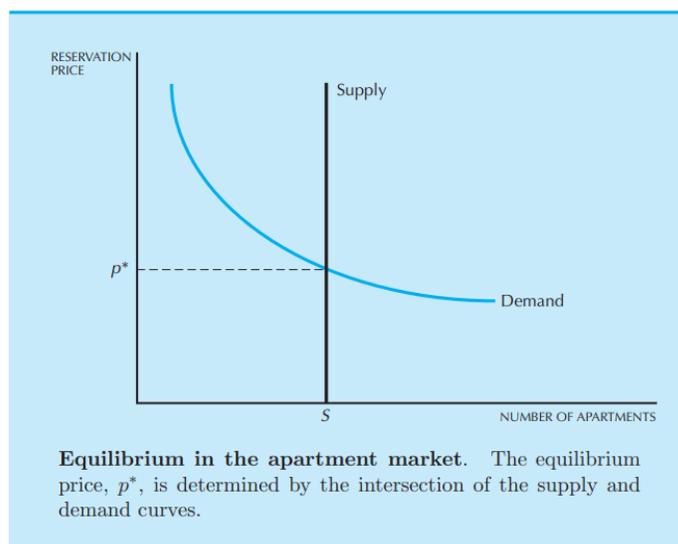


Figure 1.3: Equilibrium price  $p^*$  at the intersection of demand with short-run supply  $S$ .

### Comparative statics intuition at a glance

- Demand shift right (more students, better inner-ring amenities)  $\Rightarrow p^* \uparrow$ .
- Demand shift left (worse campus access, improved bus service)  $\Rightarrow p^* \downarrow$ .
- Short-run supply fixed: all action is in price. Long run: quantity can adjust too.

## 1.6 Comparative Statics

We study how  $p^*$  changes when fundamentals shift.

- **Demand shifters:** population, incomes, outside-option quality/cost.
- **Supply shifters (long run):** construction costs, zoning, interest rates.

**Rule of thumb:** With vertical short-run supply, *any* inward/outward shift of  $D$  maps one-for-one into lower/higher prices.

## 1.7 Other Ways to Allocate Apartments

Beyond simple price–quantity equilibrium, institutions can allocate scarce units differently. We compare efficiency and distribution.

### 1.7.1 The Discriminating Monopolist

A landlord (or central allocator) could, in principle, charge each renter exactly their reservation price, **first-degree price discrimination**. Quantity rented equals  $S$  (efficient), and all surplus is captured as revenue.

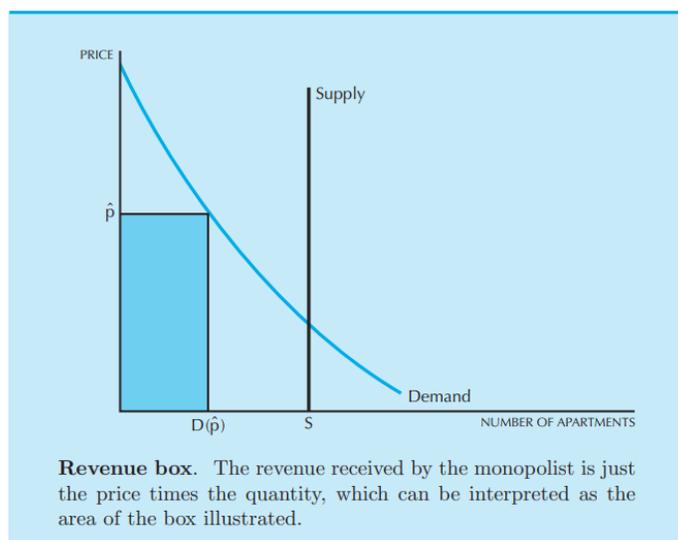


Figure 1.4: Revenue as a price–quantity box at a posted price  $\hat{p}$ . Under perfect discrimination, the monopolist extracts (almost) the entire area under demand up to  $S$ .

### 1.7.2 The Ordinary Monopolist

A single price  $\hat{p}$  maximizes revenue/profit, trading off price vs. quantity demanded. With downward-sloping  $D$ , the monopolist typically chooses  $\hat{p} > p^*$  and rents fewer than  $S$  units (if allowed to keep units vacant), creating deadweight loss relative to the competitive allocation.

### 1.7.3 Rent Control

A binding price ceiling  $p_c < p^*$  creates excess demand:

- *Short run*: price can't clear the market  $\Rightarrow$  shortages, queues, non-price rationing.
- *Long run*: reduces incentives to build/maintain units; effective supply may fall.

### 1.7.4 Which Way Is Best?

“Best” depends on the criterion:

- **Efficiency (total surplus)**: maximize sum of consumers' and producers' valuations.
- **Equity**: who receives apartments and who pays how much.

## 1.8 Pareto Efficiency

### Key Idea

An allocation is **Pareto efficient** if there is no way to make someone better off without making someone else worse off. With fixed supply and quasilinear preferences, giving each unit to the highest valuer achieves efficiency.

Pricing at  $p^*$  with vertical supply assigns inner-ring units to those with reservation value  $\geq p^*$ , which aligns with “highest-valued use.” Rent control with queues can misallocate (someone with low value gets a unit while a higher-valuing renter waits), reducing efficiency.

## 1.9 Equilibrium in the Long Run

When building/renovating is possible:

- If  $p^* >$  long-run average cost, new units enter  $\Rightarrow$  supply expands, moderating price.
- If  $p^* <$  long-run average cost, units exit or convert to other uses.

**Takeaway:** Short run = price adjusts; Long run = both price and quantity adjust as supply becomes elastic.

## 1.10 Comparing Ways to Allocate Apartments

Institution	Quantity Allocated	Efficiency / Surplus
Competitive price $p^*$	Exactly $S$ to highest valuers	Efficient in short run; surplus split
Discriminating monopolist	$S$ to highest valuers	Efficient quantity; monopolist captures (nearly) all surplus
Ordinary monopolist (single price)	Generally $< S$ if vacancies allowed	Inefficient (deadweight loss)
Rent control ( $p_c < p^*$ )	$S$ , but misallocated via queues/lotteries	Inefficient if allocation not to highest valuers; potential quality deterioration

## Chapter 2: Budget Constraint

### 2.1 Introduction

Consumers face limits on what they can afford. These limits are summarized by the **budget constraint**, which shows all combinations of goods and services a consumer can buy given prices and income.

The budget constraint provides the *economic environment* within which consumers make their choices. It reflects the trade-offs imposed by prices and income.

### 2.2 The Budget Constraint

Suppose there are two goods: good 1 and good 2. Let:

$$p_1 = \text{price of good 1}, \quad p_2 = \text{price of good 2}, \quad m = \text{consumer's income.}$$

If the consumer purchases  $x_1$  units of good 1 and  $x_2$  units of good 2, total spending must not exceed income:

$$p_1x_1 + p_2x_2 \leq m$$

The combinations  $(x_1, x_2)$  that satisfy this inequality form the **budget set**, the set of affordable bundles. The boundary of this set, where spending exactly equals income, is the **budget line**:

$$p_1x_1 + p_2x_2 = m$$

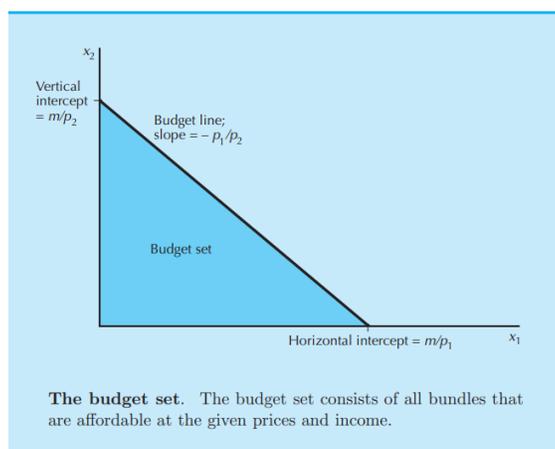


Figure 2.1: The budget set. The shaded area shows all affordable bundles. The boundary line shows bundles where income is fully spent.

### Intercepts and slope of the budget line

- Vertical intercept:  $x_2 = m/p_2$  (when all income is spent on good 2).
- Horizontal intercept:  $x_1 = m/p_1$  (when all income is spent on good 1).
- Slope:

$$\frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$$

This represents the **opportunity cost** of good 1 in terms of good 2. To get one more unit of good 1, the consumer must give up  $p_1/p_2$  units of good 2.

#### Key Idea

The slope of the budget line tells us the rate at which the market allows the consumer to trade one good for another.

## 2.3 Two Goods Are Often Enough

Although real life involves many goods, economists can usually simplify analysis to two goods without losing generality:

- Good 1 = the good of interest (e.g., housing, food, leisure).
- Good 2 = a *composite good* representing “all other goods.”

If we measure good 2 in dollars spent on “everything else,” then  $p_2 = 1$  and  $x_2$  simply equals the amount of money spent on other goods. The budget constraint becomes:

$$p_1x_1 + x_2 = m \quad \Rightarrow \quad x_2 = m - p_1x_1$$

The slope  $-p_1$  now directly represents the dollar cost of consuming more of good 1.

#### Key Idea

Reducing the world to two goods, one being “money spent on all other things”—lets us graphically represent consumer choices in two dimensions without losing the logic of optimization.

## 2.4 Properties of the Budget Set

- The budget set is **downward sloping**: to get more of one good, you must give up some of the other.
- It is **linear** if prices and income are constant.
- It is **bounded**: maximum consumption of each good is limited by income.
- The line shifts or pivots when income or prices change.

#### Example

Suppose  $m = 100$ ,  $p_1 = 5$ , and  $p_2 = 10$ . The consumer can afford any bundle satisfying  $5x_1 + 10x_2 \leq 100$ .

- All income on good 1:  $x_1 = 20, x_2 = 0$ .
- All income on good 2:  $x_2 = 10, x_1 = 0$ .
- One more unit of good 1 costs 5, which means giving up 0.5 units of good 2.

## 2.5 How the Budget Line Changes

### 2.5.1 Changing Income:

When income increases from  $m$  to  $m'$ , the intercepts increase proportionally, but the slope  $-p_1/p_2$  stays the same.

$$p_1x_1 + p_2x_2 = m' \quad \Rightarrow \quad \text{Parallel outward shift.}$$

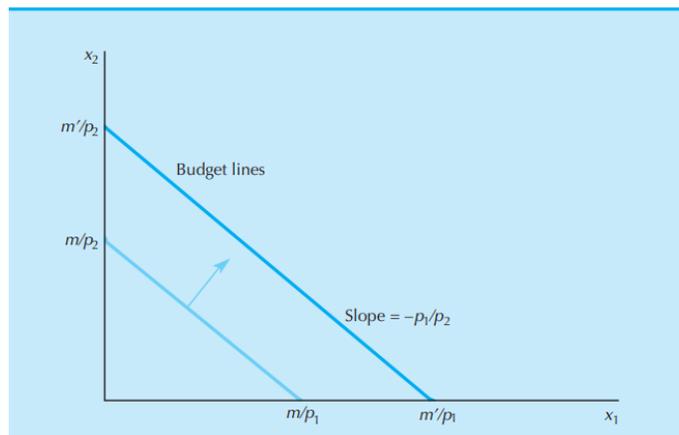


Figure 2.2: An increase in income shifts the budget line outward in a parallel way. All goods become more affordable.

**Intuition:** When income changes, purchasing power changes equally for all goods, but trade-offs (relative prices) remain the same.

### 2.5.2 Changing Prices

If the price of one good changes while income and the other price stay fixed:

- **Increase in  $p_1$ :** Budget line becomes steeper; horizontal intercept  $m/p_1$  decreases.
- **Decrease in  $p_1$ :** Budget line becomes flatter; intercept increases.

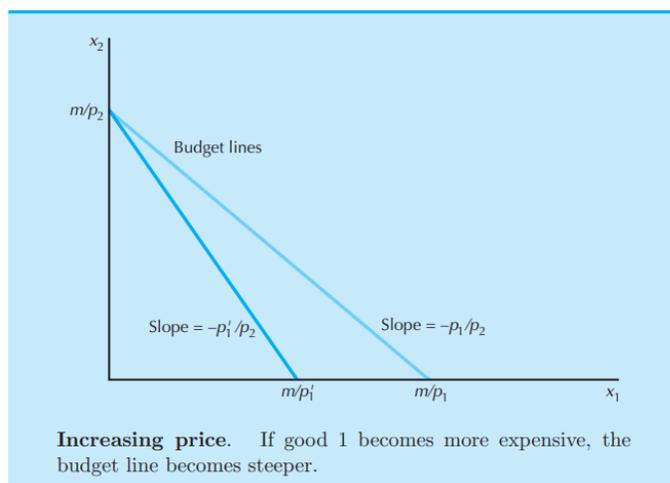


Figure 2.3: When the price of good 1 increases, the budget line pivots inward around the  $x_2$ -intercept.

### Key Idea

Changes in income cause *parallel shifts*; changes in relative prices cause *rotations*.

## 2.6 The Numeraire

In economic analysis, we often measure all prices relative to one good, called the **numeraire**. Setting the price of good 2 equal to 1 gives:

$$p_1x_1 + x_2 = m \quad \Rightarrow \quad x_2 = m - p_1x_1.$$

Here:

- $x_2$  is measured in dollars spent on “other goods.”
- $p_1$  shows how many dollars must be given up for one more unit of good 1.

Normalizing one price to 1 doesn’t affect the slope of the budget line or consumer choices—it simply changes the units of measurement.

### Key Idea

The numeraire simplifies algebra and keeps focus on relative prices, which are what actually determine behavior.

## 2.7 Taxes, Subsidies, and Rationing

Economic policy often changes the shape or position of the budget line.

### 2.7.1 Quantity Tax

If the government imposes a tax of  $t$  per unit of good 1, then the consumer pays  $p_1 + t$  per unit. The new budget constraint:

$$(p_1 + t)x_1 + p_2x_2 = m$$

The budget line becomes steeper:

$$\text{slope} = -\frac{p_1 + t}{p_2}$$

This reduces the effective purchasing power for good 1.

### 2.7.2 Quantity Subsidy

If the government gives a subsidy of  $s$  per unit of good 1, the effective price is  $p_1 - s$ :

$$(p_1 - s)x_1 + p_2x_2 = m, \quad \text{slope} = -\frac{p_1 - s}{p_2}.$$

The line becomes flatter, so good 1 becomes relatively cheaper.

### 2.7.3 Ad Valorem (Proportional) Tax or Subsidy

$$p'_1 = (1 + t)p_1 \quad \text{or} \quad p'_1 = (1 - s)p_1.$$

Changes affect only relative price ratios; the slope becomes  $-(1 + t)p_1/p_2$  or  $-(1 - s)p_1/p_2$ .

### 2.7.4 Taxes on Consumption Beyond a Threshold

If a tax applies only after  $x_1 > \bar{x}_1$ , the budget line has a *kink* at  $\bar{x}_1$ :

$$\text{slope before } \bar{x}_1 = -\frac{p_1}{p_2}, \quad \text{slope after } \bar{x}_1 = -\frac{p_1 + t}{p_2}.$$

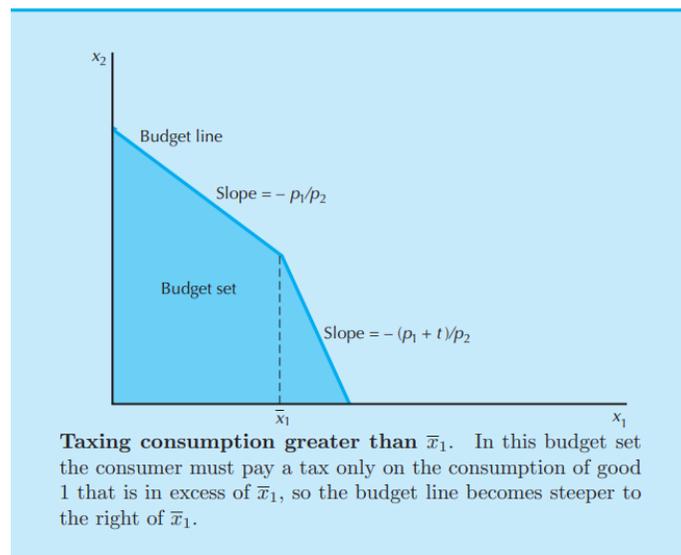


Figure 2.4: A tax on consumption beyond  $\bar{x}_1$  creates a kinked budget line.

### 2.7.5 Rationing

If the consumer cannot buy more than  $\bar{x}_1$  units of good 1, the budget set is truncated:

$$p_1x_1 + p_2x_2 \leq m, \quad x_1 \leq \bar{x}_1.$$

Beyond  $\bar{x}_1$ , extra consumption is forbidden regardless of income.

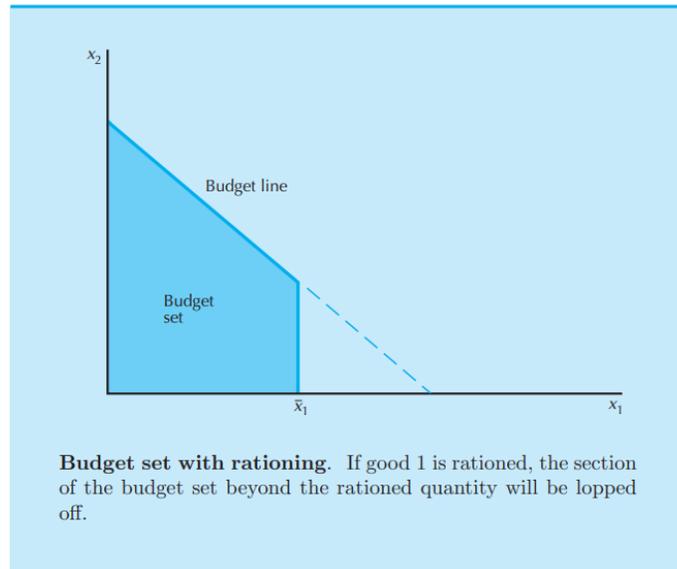


Figure 2.5: Rationing good 1: part of the budget set is cut off beyond  $\bar{x}_1$ .

#### Key Idea

Taxes and subsidies change slopes (relative prices). Rationing cuts off part of the feasible set. Lump-sum transfers move the line parallelly.

## 2.8 Budget Line Changes (Summary and Geometry)

- **Income change:** Parallel shift (same slope).
- **Price change:** Pivot around the intercept.
- **Tax/subsidy:** Change in slope (steeper or flatter).
- **Rationing:** Kink or cutoff.
- **Threshold tax:** Two-part slope.

#### Key Idea

All of these can be viewed as shifts or rotations of the line

$$p_1x_1 + p_2x_2 = m,$$

which define the consumer's opportunity set.

## Chapter 3: Preferences

### 3.1 Introduction

Consumer preferences describe how individuals rank different bundles of goods. The study of preferences forms the foundation of consumer theory, it explains how people make choices given their tastes and constraints.

We assume that consumers are **rational**, meaning they can compare bundles and make consistent decisions that reveal what they prefer.

#### Key Idea

A consumer's preferences determine which bundles they like better, worse, or equally well. These preferences, together with the budget constraint, determine their optimal choice.

### 3.2 Consumer Preferences

A **bundle** is a combination of goods  $(x_1, x_2)$  where  $x_1$  and  $x_2$  represent quantities of two goods.

We use the following notation:

- $(x_1, x_2) \succ (y_1, y_2)$  means the consumer **strictly prefers** bundle  $(x_1, x_2)$  to bundle  $(y_1, y_2)$ .
- $(x_1, x_2) \sim (y_1, y_2)$  means the consumer is **indifferent** between the two bundles.
- $(x_1, x_2) \succeq (y_1, y_2)$  means the consumer **weakly prefers**  $(x_1, x_2)$  to  $(y_1, y_2)$ , that is, either strictly prefers it or is indifferent.

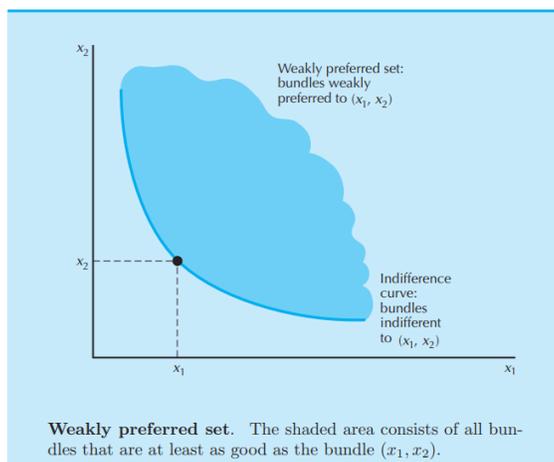


Figure 3.1: The weakly preferred set. The shaded region shows all bundles that are at least as good as  $(x_1, x_2)$ .

### 3.3 Assumptions about Preferences

Economists make several standard assumptions about consumer preferences:

#### 1. Completeness

For any two bundles  $(x_1, x_2)$  and  $(y_1, y_2)$ , the consumer can always state a preference:

$$(x_1, x_2) \succeq (y_1, y_2), \quad (y_1, y_2) \succeq (x_1, x_2), \quad \text{or both.}$$

Consumers can rank all bundles.

#### 2. Transitivity

If  $(x_1, x_2) \succeq (y_1, y_2)$  and  $(y_1, y_2) \succeq (z_1, z_2)$ , then

$$(x_1, x_2) \succeq (z_1, z_2).$$

Preferences are logically consistent, no circular reasoning.

#### 3. Monotonicity (More is Better)

If a bundle has at least as much of both goods and more of one, it is strictly preferred:

$$x_1 \geq y_1, \quad x_2 \geq y_2, \quad \text{and at least one strict inequality} \Rightarrow (x_1, x_2) \succ (y_1, y_2).$$

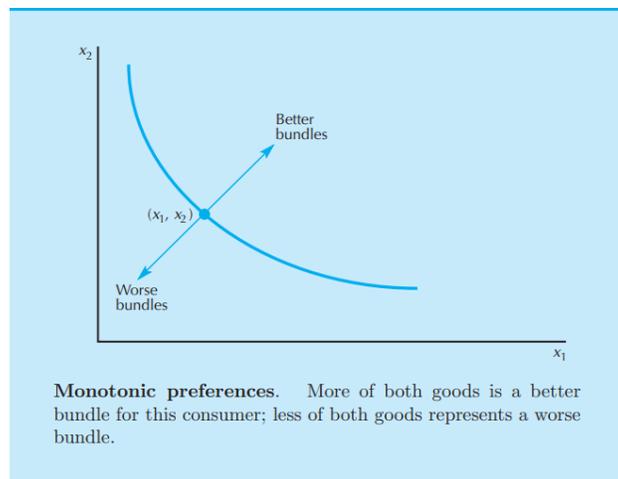


Figure 3.2: Monotonic preferences: more of both goods leads to a better bundle.

#### 4. Convexity (Averages are Preferred to Extremes)

Consumers prefer balanced bundles to extreme ones:

$$(x_1, x_2) \sim (y_1, y_2) \Rightarrow t(x_1, x_2) + (1 - t)(y_1, y_2) \succeq (y_1, y_2),$$

for all  $t \in [0, 1]$ .

This means that indifference curves are **bowed toward the origin** (convex).

## 5. Continuity

Small changes in goods cause only small changes in preferences; there are no jumps or gaps.

### Key Idea

These assumptions make preferences **well-behaved**: complete, transitive, monotonic, convex, and continuous. They ensure that an optimal consumption bundle exists and is unique.

## 3.4 Indifference Curves

An **indifference curve** represents all bundles of goods that give the same level of satisfaction (utility).

$$(x_1, x_2) \sim (y_1, y_2)$$

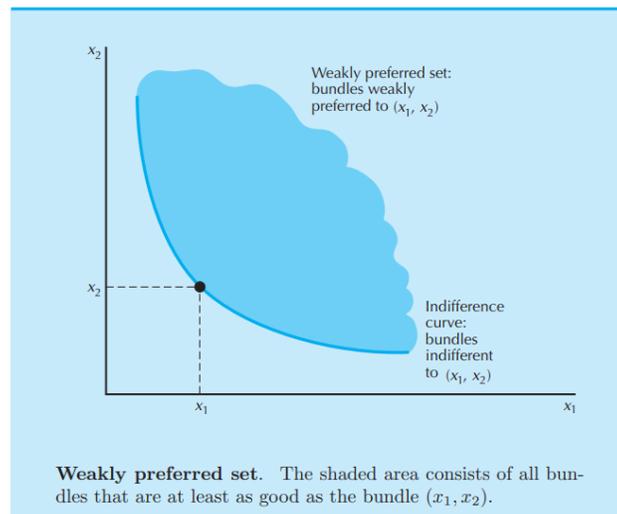


Figure 3.3: An indifference curve and the weakly preferred set.

### Properties of Indifference Curves

- They are **downward sloping**: more of one good compensates for less of another.
- They cannot cross (violates transitivity).
- Higher curves represent higher satisfaction.
- They are usually convex to the origin (due to diminishing MRS).

## 3.5 Examples of Preferences

### 3.5.1 Perfect Substitutes

Goods that the consumer views as identical in use. Example: red and blue pencils.

$$u(x_1, x_2) = ax_1 + bx_2$$

Indifference curves are straight lines with constant slope  $-a/b$ .

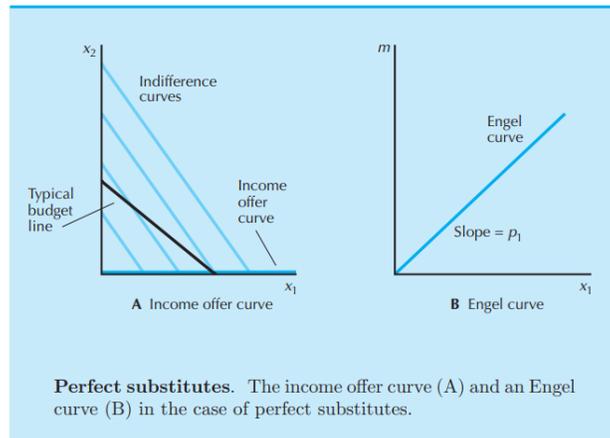


Figure 3.4: Perfect substitutes: straight-line indifference curves.

### 3.5.2 Perfect Complements

Goods that are always consumed in fixed proportions. Example: left and right shoes.

$$u(x_1, x_2) = \min\{ax_1, bx_2\}$$

Indifference curves are **L-shaped**.

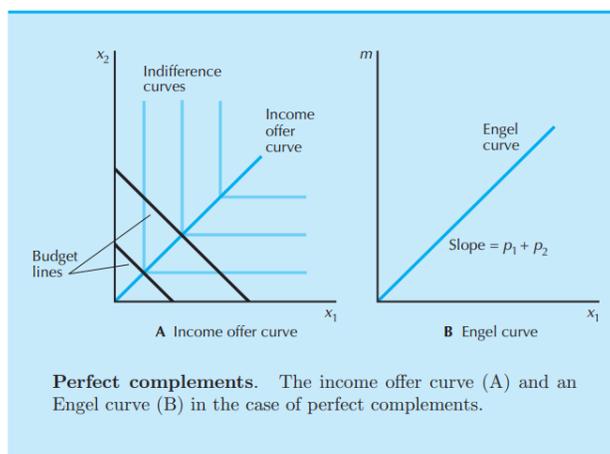


Figure 3.5: Perfect complements: L-shaped indifference curves.

### 3.5.3 Bads

A “bad” is something the consumer dislikes. Example: pollution or anchovies. Indifference curves slope upward because to compensate for more of the bad, the consumer must also get more of the good.

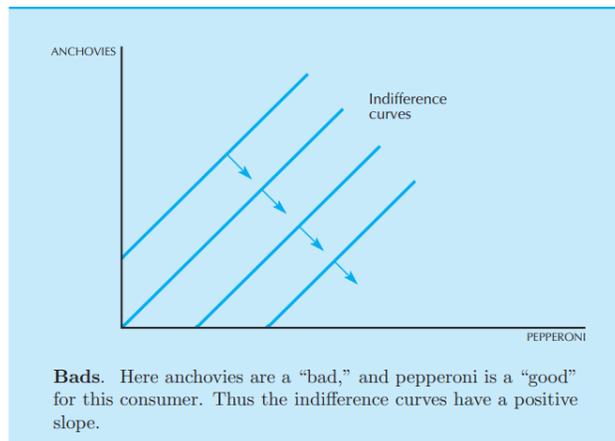


Figure 3.6: A bad: indifference curves slope upward (more bad requires more good).

### 3.5.4 Neutral Good

A good that the consumer does not care about. Example: anchovies if you are indifferent to them.

Utility depends only on the other good.

Indifference curves are vertical (if good 2 is neutral).

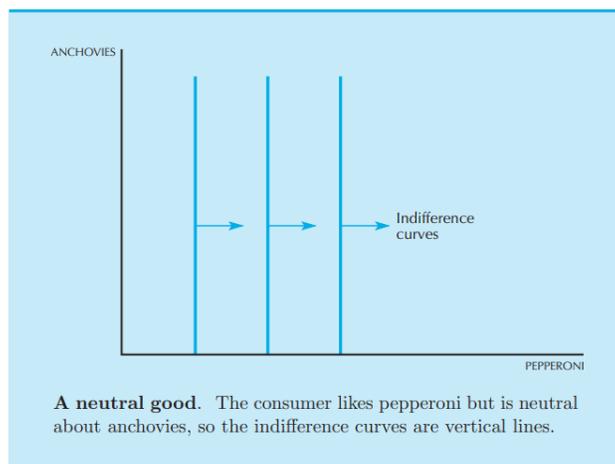


Figure 3.7: A neutral good: indifference curves are vertical.

### 3.5.5 Satiation (Bliss Point)

There exists a most-preferred bundle  $(\bar{x}_1, \bar{x}_2)$ , called the **satiation point**.

Utility decreases as consumption moves away from this point.

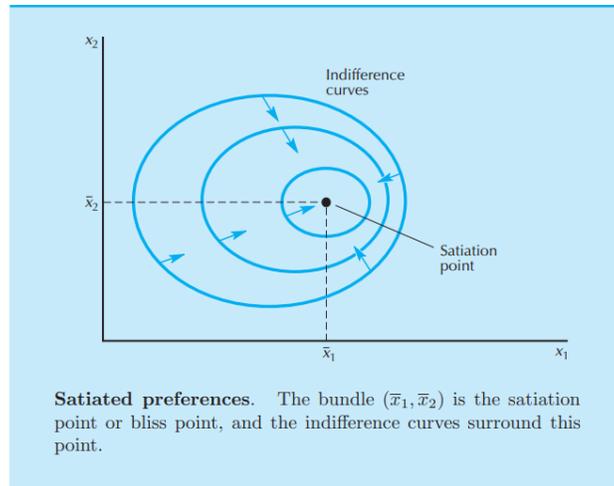


Figure 3.8: Satiated preferences: indifference curves form concentric rings around the bliss point.

### 3.5.6 Discrete Goods

Some goods can only be consumed in whole units (like cars or apartments). Indifference curves appear as a set of discrete points rather than continuous curves.

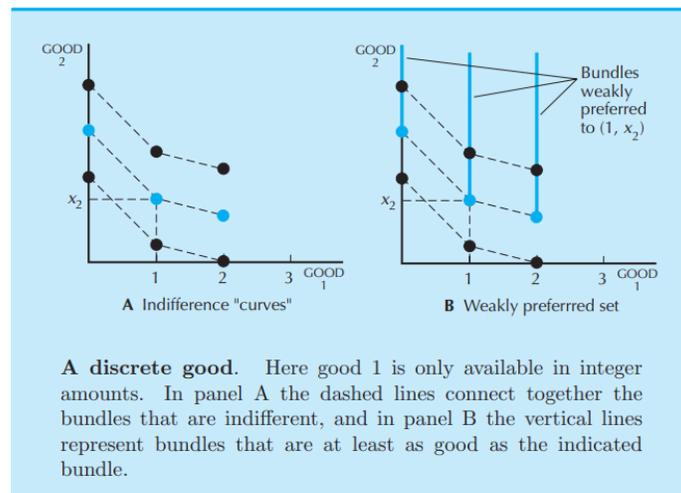


Figure 3.9: A discrete good: bundles and weakly preferred sets in integer quantities.

## 3.6 Well-Behaved Preferences

Preferences are **well-behaved** if they are:

1. **Monotonic:** more of both goods is preferred to less.
2. **Convex:** averages are preferred to extremes.

**Result:** Well-behaved preferences produce smooth, downward-sloping, and convex indifference curves that never cross.

**Key Idea**

Well-behaved preferences guarantee that consumers will choose an interior optimum, where their budget line just touches the highest attainable indifference curve.

**3.7 The Marginal Rate of Substitution (MRS)**

The **Marginal Rate of Substitution (MRS)** measures how much of good 2 the consumer is willing to give up for one additional unit of good 1 while keeping utility constant.

$$MRS = -\frac{\Delta x_2}{\Delta x_1} \quad \text{or (in calculus form)} \quad MRS = -\frac{MU_1}{MU_2}$$

where  $MU_1$  and  $MU_2$  are the marginal utilities of goods 1 and 2.

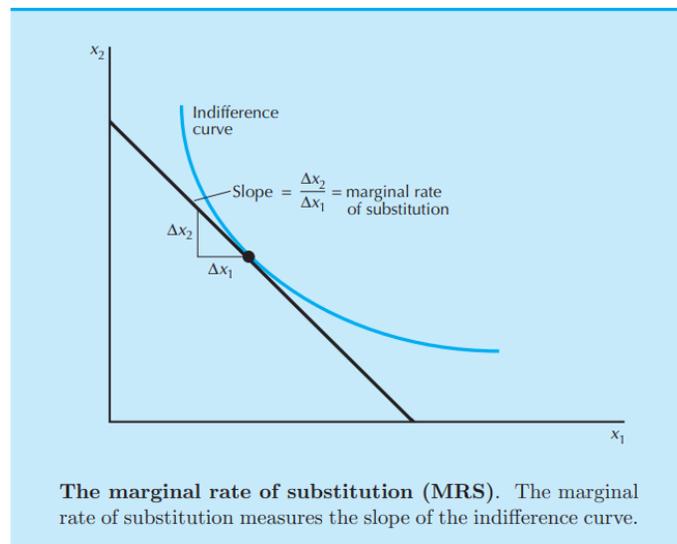


Figure 3.10: Marginal Rate of Substitution: slope of the indifference curve.

**Interpretation:** -  $MRS$  is the *subjective trade-off rate*. - As we move down an indifference curve,  $MRS$  usually decreases, this is called **diminishing MRS**.

**Key Idea**

At the optimal choice, the consumer's  $MRS$  equals the market trade-off given by prices:

$$MRS = \frac{p_1}{p_2}$$

**3.8 Other Interpretations of MRS**

- The  $MRS$  shows how much the consumer is *willing to pay* for an additional unit of one good in terms of the other.

- It reflects the slope of the consumer's indifference curve at a particular point.
- It connects subjective preferences (utility) to objective market trade-offs (prices).

$$MRS = \frac{MU_1}{MU_2} = \frac{\text{willingness to give up good 2}}{\text{gain from one more unit of good 1}}$$

### 3.9 Behavior of the MRS

- **Diminishing MRS:** As the consumer consumes more of good 1, they are willing to give up less of good 2 for additional units of good 1.
- **Convex indifference curves:** Reflects a diminishing MRS, trade-offs become less extreme as the consumer moves along the curve.
- **Linear indifference curves:** Constant MRS (perfect substitutes).
- **L-shaped indifference curves:** Undefined MRS along the kink (perfect complements).

#### Key Idea

The shape of the indifference curve reveals how flexible the consumer is in substituting between goods. The more curved the indifference curve, the less willing the consumer is to substitute one good for another.

## Chapter 4: Utility

### 4.1 Introduction

Utility represents a consumer's satisfaction or happiness from consuming goods and services. It provides a numerical way to represent preferences, allowing us to compare bundles mathematically. Although utility itself is not directly measurable, it is a convenient tool to describe *ordinal preferences* (ranking of bundles).

#### Example

Utility is a way to represent preferences numerically. It doesn't measure actual happiness, it only helps rank choices.

### 4.2 Cardinal vs. Ordinal Utility

#### Ordinal Utility

- Only the **ranking** of preferences matters, not the absolute numbers.
- Example: If  $U(A) = 10$  and  $U(B) = 5$ , bundle  $A$  is preferred to  $B$ , but it doesn't mean  $A$  is twice as good.

#### Cardinal Utility

- Implies the actual magnitude of utility differences has meaning.
- For most modern economics, we only use utility as an **ordinal concept**.

#### Example

In consumer theory, utility functions are ordinal: any monotonic transformation of a utility function represents the same preferences.

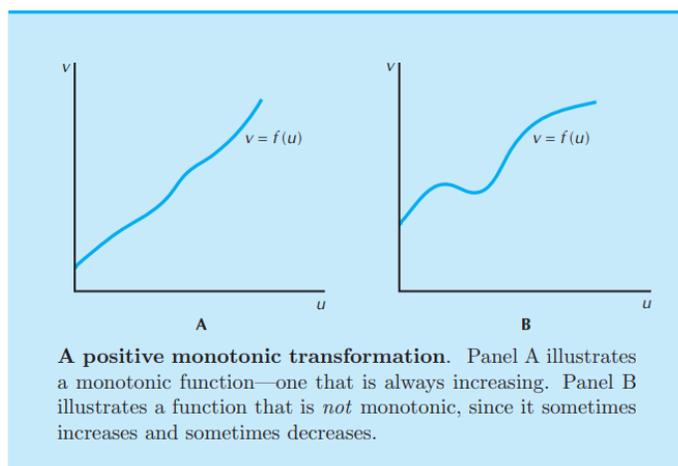


Figure 4.1: Positive monotonic transformations. Panel A: monotonic (always increasing) function; Panel B: not monotonic.

### 4.3 Constructing a Utility Function

Given well-behaved preferences, we can assign a utility function  $u(x_1, x_2)$  such that:

$$(x_1, x_2) \succeq (y_1, y_2) \quad \text{if and only if} \quad u(x_1, x_2) \geq u(y_1, y_2).$$

- Any function that preserves this ordering represents the same preferences.
- Monotonic transformations like  $\tilde{u}(x_1, x_2) = f(u(x_1, x_2))$  (where  $f' > 0$ ) describe the same indifference curves.

**Interpretation:** Utility functions are not unique; many different mathematical forms can describe the same preference structure.

#### Example

Utility assigns numbers to bundles so that higher numbers correspond to more preferred bundles. Only the order matters, not the scale.

### 4.4 Some Common Utility Functions

#### 4.4.1 Perfect Substitutes

When two goods are viewed as identical in satisfaction:

$$u(x_1, x_2) = ax_1 + bx_2$$

Indifference curves are straight lines with constant slope  $-a/b$ . **Example:** red and blue pencils.

### 4.4.2 Perfect Complements

Goods consumed in fixed proportions:

$$u(x_1, x_2) = \min\{ax_1, bx_2\}$$

Indifference curves are L-shaped, reflecting that extra amounts of one good alone add no utility.

**Example:** left and right shoes.

### 4.4.3 Cobb–Douglas Preferences

A widely used smooth utility function:

$$u(x_1, x_2) = x_1^c x_2^d, \quad c, d > 0$$

Each good is essential; preferences are convex and exhibit diminishing MRS.

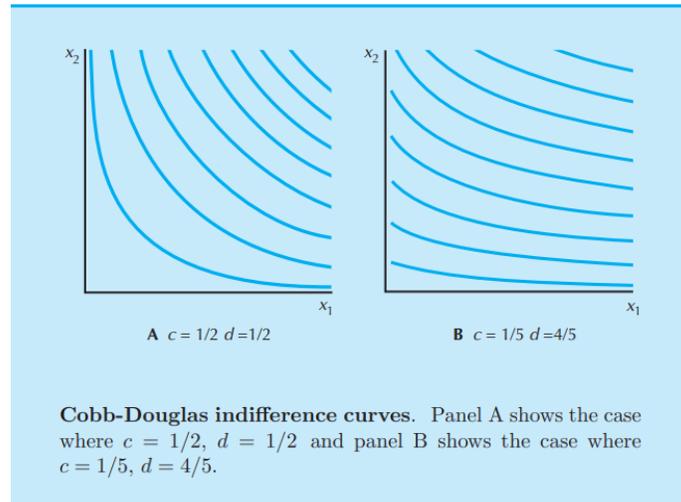


Figure 4.2: Cobb–Douglas indifference curves. Panel A: equal weights ( $c = d = 1/2$ ); Panel B: stronger preference for good 2 ( $c = 1/5$ ,  $d = 4/5$ ).

#### Example

In Cobb–Douglas preferences, consumers always spend fixed shares of income on each good, independent of prices or income level.

### 4.4.4 Quasilinear Preferences

Utility is linear in one good but nonlinear in the other:

$$u(x_1, x_2) = v(x_1) + x_2$$

- Indifference curves are vertically parallel shifts of one another.
- The MRS depends only on  $x_1$ .

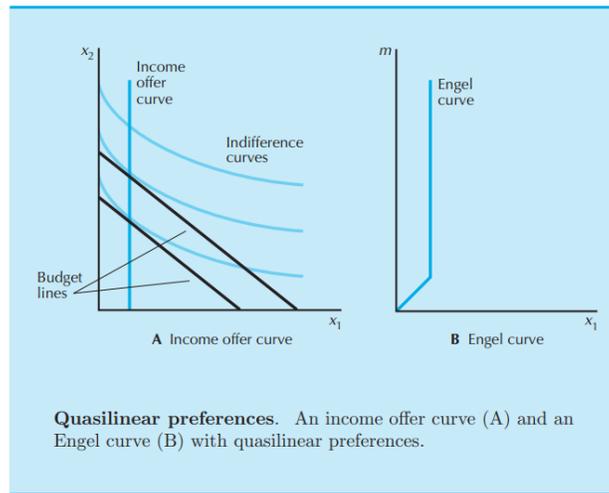


Figure 4.3: Quasilinear preferences: vertically shifted indifference curves.

### 4.4.5 Example: Indifference Curves from Utility

For the Cobb–Douglas example  $u(x_1, x_2) = x_1^c x_2^d$ , a given utility level  $\bar{u}$  implies:

$$x_2 = \left( \frac{\bar{u}}{x_1^c} \right)^{1/d}$$

Each indifference curve corresponds to a fixed  $\bar{u}$ .

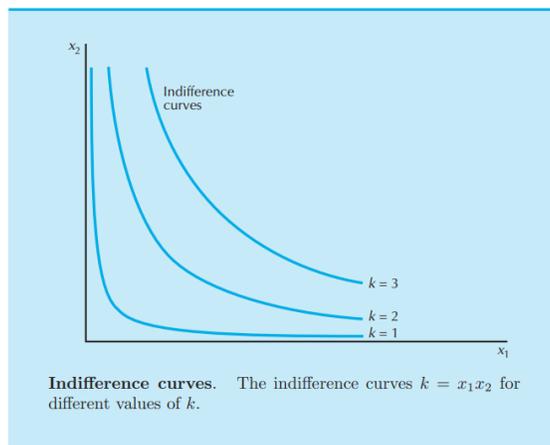


Figure 4.4: Indifference curves  $k = x_1 x_2$  for different values of  $k$ .

## 4.5 Marginal Utility

The **marginal utility** (MU) of a good measures how utility changes with a small increase in that good, holding the other good constant.

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1}, \quad MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2}$$

### Interpretation:

- $MU_1$  = extra satisfaction from one more unit of good 1.
- $MU_2$  = extra satisfaction from one more unit of good 2.

### Example

For the Cobb–Douglas utility  $u(x_1, x_2) = x_1^c x_2^d$ :

$$MU_1 = c x_1^{c-1} x_2^d, \quad MU_2 = d x_1^c x_2^{d-1}.$$

### Key Idea

Marginal utility is positive but typically diminishing, the more of a good you have, the smaller the additional satisfaction from consuming another unit.

## 4.6 Marginal Utility and the MRS

The **marginal rate of substitution** (MRS) equals the ratio of marginal utilities:

$$MRS = -\frac{MU_1}{MU_2}$$

It measures how many units of good 2 the consumer is willing to give up for one extra unit of good 1, keeping utility constant.

### Key Idea

For  $u(x_1, x_2) = x_1^c x_2^d$ ,

$$MRS = -\frac{c x_2}{d x_1}.$$

As  $x_1$  increases (and  $x_2$  decreases),  $MRS$  diminishes, reflecting convex preferences.

### Key Idea

At the optimal bundle:

$$MRS = \frac{p_1}{p_2}$$

—that is, the subjective trade-off equals the market trade-off.

## 4.7 Utility and Real-World Applications: Commuting Example

Consider two “goods”:

- $x_1$  = amount of leisure time
- $x_2$  = income (or goods purchased)

A longer commute reduces leisure (good 1) but may increase income (good 2). The consumer's indifference curves represent trade-offs between time and income. Policies such as work-from-home or transport improvements effectively shift the feasible set.

### Key Idea

Utility analysis helps us understand trade-offs in real-world decisions: not just consumption of goods, but time, convenience, and quality of life.

## Chapter 5: Choice

### 5.1 Introduction

Consumers face a trade-off between goods and must decide how to allocate their limited income among them. The central assumption of microeconomics is that individuals make choices that **maximize their utility** subject to their **budget constraint**. In this chapter, we put together the tools from the previous chapters, the budget line and indifference curves to analyze optimal consumer choice.

#### Key Idea

The optimal bundle is the combination of goods that gives the consumer the highest possible utility while still lying within the budget constraint.

The decision process involves two elements:

1. **Preferences:** determine which bundles are more or less preferred.
2. **Budget constraint:** determines which bundles are affordable.

The point of tangency between the highest attainable indifference curve and the budget line represents the consumer's **optimal choice**.

### 5.2 The Optimal Choice

The optimal bundle  $(x_1^*, x_2^*)$  must satisfy two conditions:

- It lies on the **budget line**: all income is spent.
- It lies on the **highest possible indifference curve**: utility is maximized given the budget.

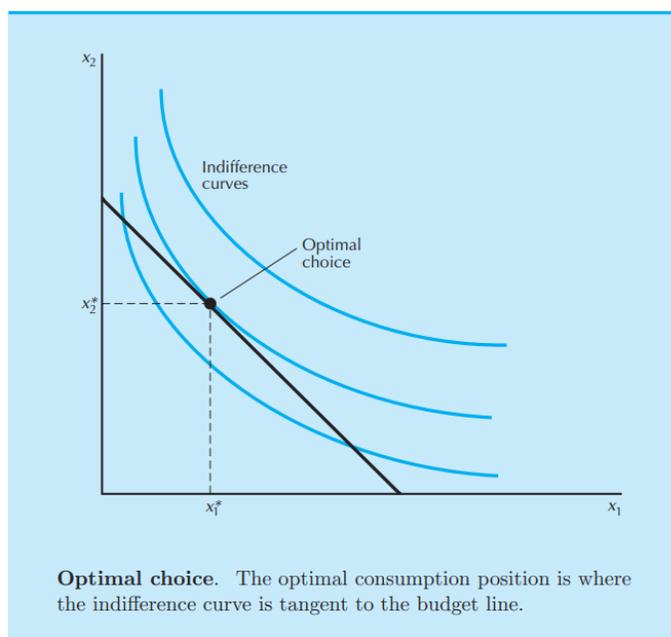


Figure 5.1: Optimal choice occurs where the indifference curve is tangent to the budget line.

At the point of tangency:

$$MRS = \frac{p_1}{p_2}$$

This means that the rate at which the consumer is willing to trade good 2 for good 1 (the slope of the indifference curve) equals the rate at which the market allows them to trade (the slope of the budget line).

### Key Idea

The **tangency condition** ensures that the consumer cannot increase utility by reallocating expenditure. If  $MRS > p_1/p_2$ , the consumer values good 1 more than the market does and should buy more of good 1. If  $MRS < p_1/p_2$ , they should buy less of good 1.

### Mathematical Derivation (Lagrangian Approach)

The consumer maximizes utility:

$$\max_{x_1, x_2} U(x_1, x_2)$$

subject to:

$$p_1x_1 + p_2x_2 = m$$

The Lagrangian function is:

$$\mathcal{L} = U(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2)$$

Taking first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial x_1} = U_1 - \lambda p_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = U_2 - \lambda p_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = m - p_1 x_1 - p_2 x_2 = 0$$

From the first two equations:

$$\frac{U_1}{U_2} = \frac{p_1}{p_2}$$

$$\Rightarrow MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

and the budget constraint gives  $p_1 x_1 + p_2 x_2 = m$ . Together, these determine  $(x_1^*, x_2^*)$ .

### Key Idea

At the optimal bundle, the **marginal utility per dollar spent** is the same for all goods:

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

Otherwise, the consumer could increase total utility by shifting spending toward the good with higher utility per dollar.

## 5.3 Kinky and Boundary Solutions

### 5.3.1 Kinky Preferences

Sometimes, indifference curves are not smooth but have “kinks” (as with perfect complements). At the kink, there is no single tangent slope. The optimal choice is where the kink touches the budget line.

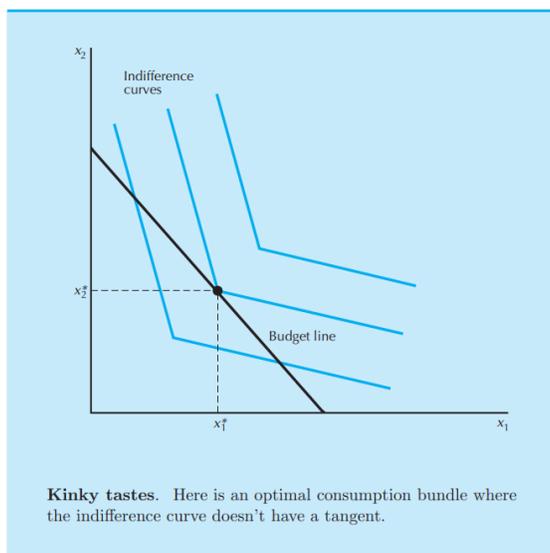


Figure 5.2: Kinky tastes: the optimal consumption bundle occurs at the kink, where no single tangent exists.

### 5.3.2 Boundary Optimum

Sometimes, the optimal choice occurs at a **corner** or **boundary** where one good is not consumed at all. This happens when the slope of the indifference curve never equals the slope of the budget line.

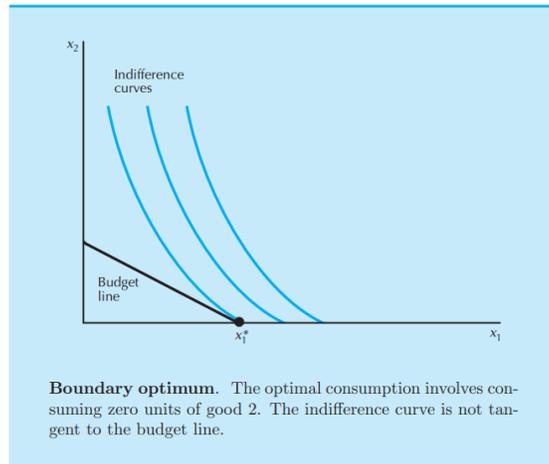


Figure 5.3: Boundary optimum: the consumer spends all income on good 1 and none on good 2.

#### Key Idea

If a consumer strongly prefers coffee to tea, they may spend all income on coffee even if tea is cheap. Here,  $x_2^* = 0$ , and the indifference curve is not tangent to the budget line.

## 5.4 Special Cases of Optimal Choice

### 5.4.1 Perfect Substitutes

If two goods are perfect substitutes, the indifference curves are straight lines. The consumer will buy only the cheaper good.

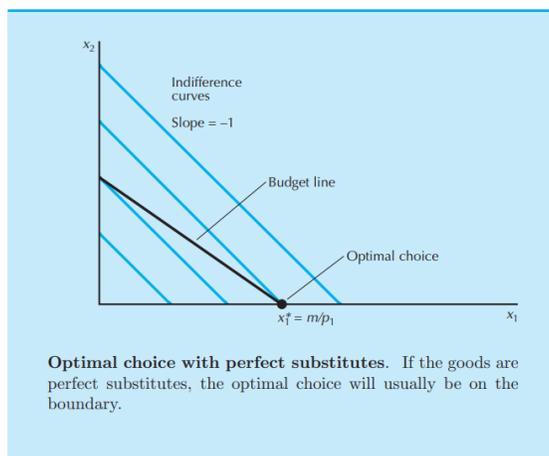


Figure 5.4: Optimal choice with perfect substitutes: the consumer buys only the cheaper good.

If  $U(x_1, x_2) = ax_1 + bx_2$ , then:

If  $\frac{a}{p_1} > \frac{b}{p_2}$ , consume only good 1. If  $\frac{a}{p_1} < \frac{b}{p_2}$ , consume only good 2.

### 5.4.2 Perfect Complements

If goods are perfect complements, the consumer wants to consume them in fixed proportions.

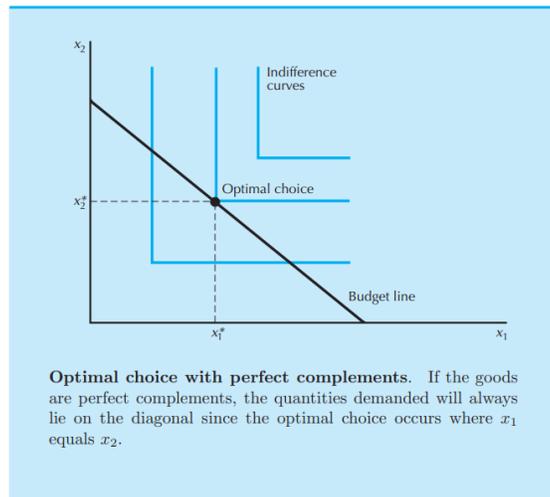


Figure 5.5: Optimal choice with perfect complements: goods are consumed in fixed ratios.

If  $U(x_1, x_2) = \min\{ax_1, bx_2\}$ , then the optimal bundle satisfies:

$$ax_1 = bx_2$$

$$\Rightarrow x_1^* = \frac{b}{a}x_2^*$$

### 5.4.3 Discrete Goods

If a good is available only in integer amounts, the consumer chooses the discrete bundle that gives the highest utility among the affordable ones.

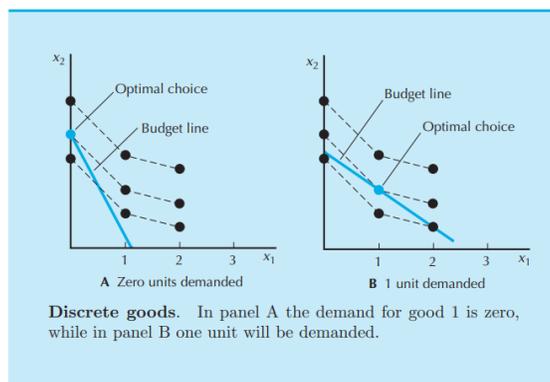


Figure 5.6: Discrete goods: choice depends on the highest attainable discrete indifference point.

## 5.5 Concave Preferences and Multiple Tangencies

### 5.5.1 Concave Preferences

For **concave preferences**, consumers prefer extremes rather than mixtures. In this case, the tangency point may not be optimal—the boundary point yields higher utility.

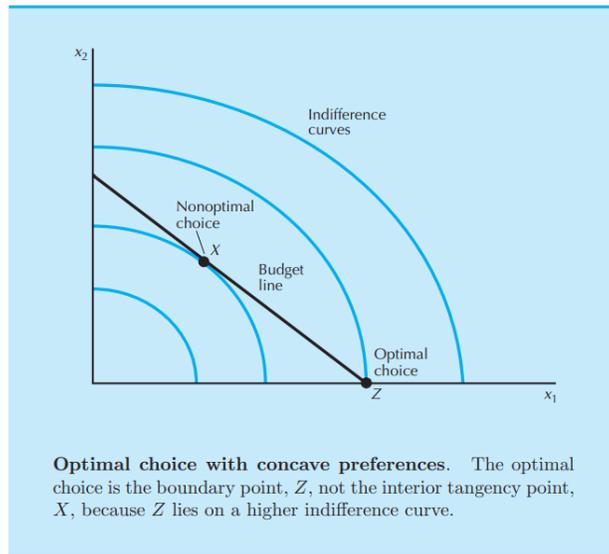


Figure 5.7: Optimal choice with concave preferences: the boundary bundle is preferred to the interior tangency.

### 5.5.2 More Than One Tangency

The tangency condition  $MRS = p_1/p_2$  is *necessary* for an optimum but not always *sufficient*. If preferences are not convex, multiple tangencies may exist, but only those on the highest indifference curve are true optima.

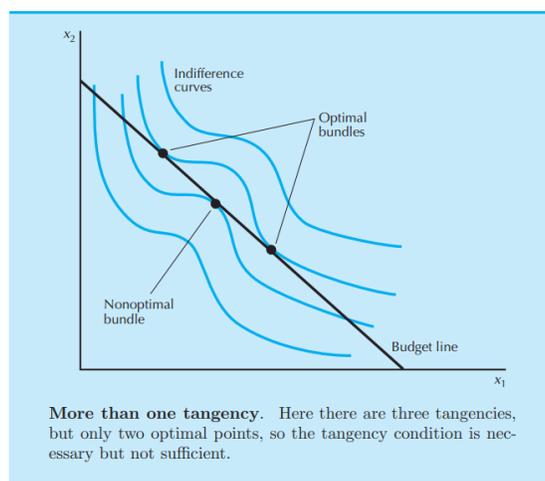


Figure 5.8: Multiple tangencies: only the highest one is truly optimal.

**Key Idea**

Convex preferences guarantee a unique tangency solution that maximizes utility. Without convexity, tangency alone does not ensure optimality.

**5.6 Interpreting the Consumer's Choice**

At the optimum  $(x_1^*, x_2^*)$ , the following conditions hold:

$$MRS = \frac{p_1}{p_2}, \quad p_1x_1^* + p_2x_2^* = m$$

From this, we can derive the **demand functions**:

$$x_1^* = x_1(p_1, p_2, m), \quad x_2^* = x_2(p_1, p_2, m)$$

These functions describe how the consumer's choice responds to changes in prices and income.

**Key Idea**

If the price of good 1 rises, the budget line pivots inward. The consumer substitutes away from good 1 (substitution effect) and may reduce overall consumption (income effect). We explore these effects formally in later chapters.

**5.7 Summary of Key Results**

- Consumers choose the bundle on the budget line that gives the highest utility.
- The tangency condition  $MRS = \frac{p_1}{p_2}$  defines the interior optimum.
- Corner or boundary solutions arise when preferences are not strictly convex.
- For perfect substitutes, consumers choose the cheaper good.
- For perfect complements, goods are consumed in fixed proportions.
- The Lagrangian method provides a general way to solve utility maximization problems.
- The solution yields individual demand functions for each good.

**Key Idea**

The essence of consumer choice: *Utility maximization subject to a budget constraint*. The equality between the marginal rate of substitution and the price ratio ensures that no reallocation can make the consumer better off.

# Chapter 6: Demand

## Introduction

The demand function shows how a consumer's optimal choice of goods changes when prices or income change. Understanding demand helps economists and policymakers predict how consumers respond to price changes, design taxation policies, measure welfare, and interpret market outcomes.

We study how to derive the **demand curve** from utility maximization, analyze how **price changes** (through substitution and income effects) alter consumption, and how **income changes** generate **income offer curves** and **Engel curves**.

Finally, we look at specific demand patterns under different preference structures, such as **Cobb–Douglas**, **perfect substitutes**, **perfect complements**, and **quasilinear preferences**, and identify cases like **ordinary goods**, **inferior goods**, and the rare **Giffen goods**.

This chapter bridges individual optimization with the market-level demand relationship, laying the groundwork for welfare analysis, elasticity measurement, and consumer surplus in later chapters.

## 6.1 From Optimal Choice to Demand

Given prices  $(p_1, p_2)$  and income  $m$ , a consumer's optimal bundle  $(x_1^*, x_2^*)$  solves

$$\max_{x_1, x_2} U(x_1, x_2) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 = m.$$

The **(Marshallian) demand function** for good  $i$  maps  $(p_1, p_2, m)$  into the chosen quantity:

$$x_i = x_i(p_1, p_2, m), \quad i = 1, 2.$$

### Key Idea

Fix  $p_2, m$  and vary  $p_1$ . The locus of optimal bundles in  $(x_1, x_2)$ -space is the **price offer curve**. Plot the corresponding  $(p_1, x_1)$  pairs and you obtain the **(own-price) demand curve** for good 1.

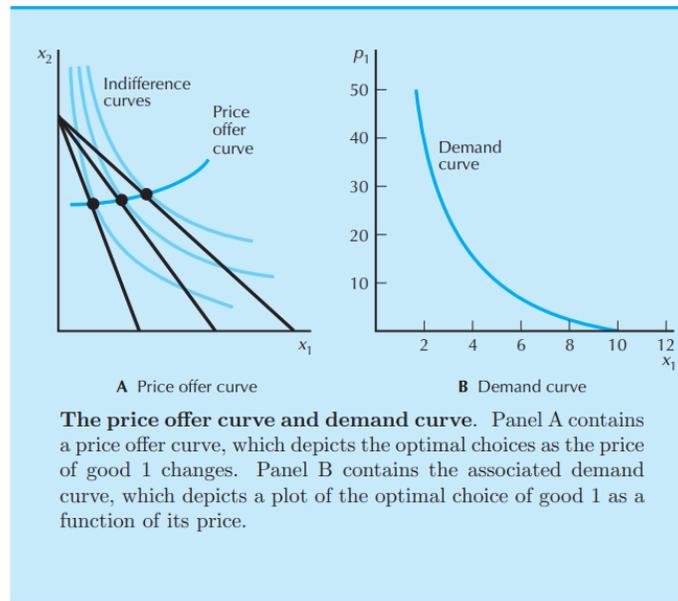


Figure 6.1: Left: price offer curve (POC) as  $p_1$  varies. Right: demand curve  $x_1(p_1)$  induced by the POC.

Sometimes it is convenient to write **inverse demand**, the price required to induce quantity  $x_1$ :

$$p_1 = p_1(x_1; p_2, m).$$

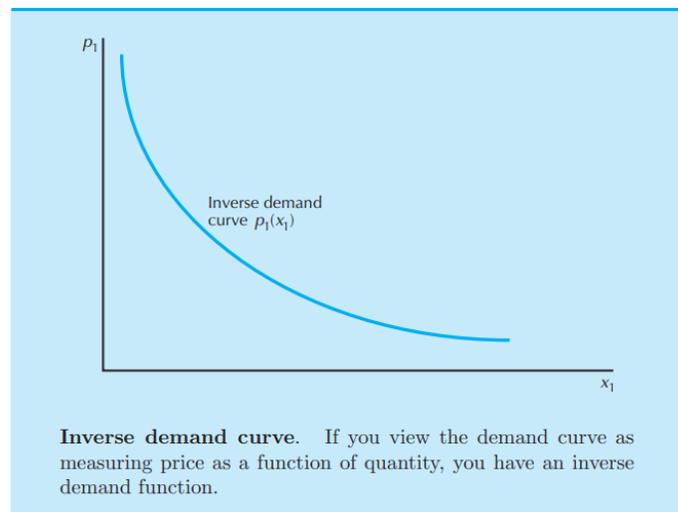
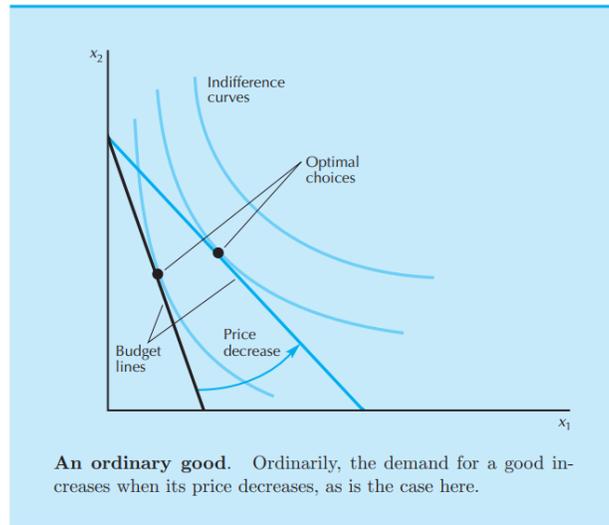


Figure 6.2: Inverse demand: price as a function of quantity.

## 6.2 Price Changes: Ordinary vs. Giffen

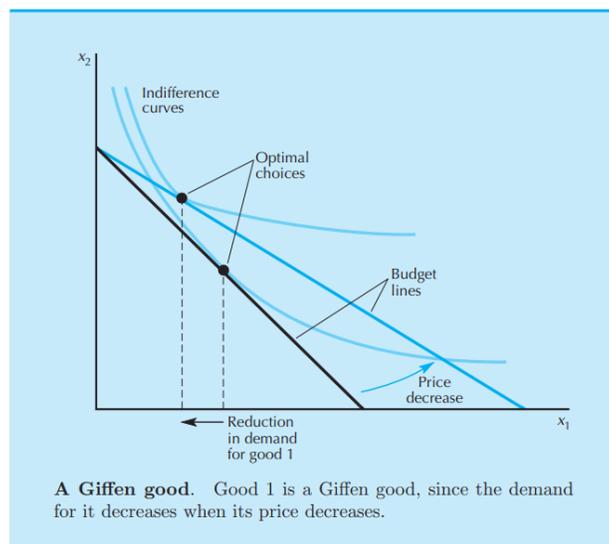
### Law of demand (typical case)

Holding  $p_2, m$  fixed, a decrease in  $p_1$  usually increases  $x_1$  (**ordinary good w.r.t. price**). Graphically: budget rotates out and the new optimum has more  $x_1$ .

Figure 6.3: Ordinary good: when  $p_1$  falls, demanded  $x_1$  rises.

### Giffen good (rare)

For some extreme preferences with strong income effects at low consumption, a fall in  $p_1$  can *reduce*  $x_1$ ; the demand curve slopes upward over a range.

Figure 6.4: Giffen case:  $x_1$  falls when  $p_1$  falls.

### Key Idea

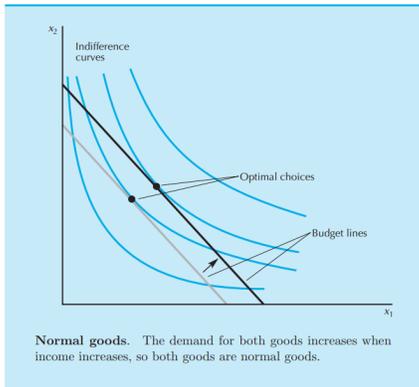
A price change has two components: (i) a **substitution effect** (always moves  $x_1$  opposite to the price change) and (ii) an **income effect** (moves  $x_1$  with or against the price change depending on whether the good is normal or inferior). A Giffen good is an inferior good with an income effect so strong that it overwhelms substitution.

### 6.3 Income Changes: Income Offer Curves and Engel Curves

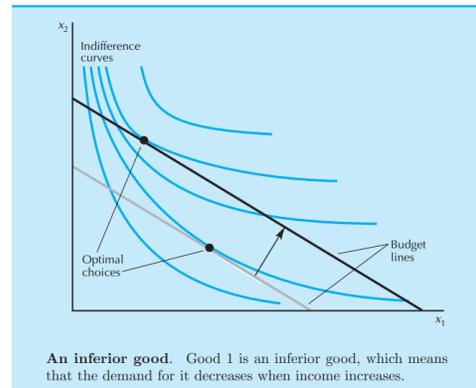
Fix prices and vary income  $m$ .

- The **income offer curve (IOC)** traces optimal bundles as  $m$  changes.
- The **Engel curve** plots  $m$  on the vertical axis and  $x_i$  on the horizontal axis.

If  $x_1$  increases with  $m$ , it is **normal**; if it decreases with  $m$ , it is **inferior**.



(a) Normal goods: as income rises, both goods' demands increase.



(b) Inferior good: as income rises, demanded  $x_1$  falls.

### 6.4 Worked-Out Utility Examples (Demand, IOC, Engel)

#### 6.4.1 Perfect Substitutes

Let  $U(x_1, x_2) = ax_1 + bx_2$  with  $a, b > 0$ . The consumer buys the cheapest “effective” good:

$$\frac{a}{p_1} \geq \frac{b}{p_2} \Rightarrow x_1^* = \begin{cases} \frac{m}{p_1}, & \frac{a}{p_1} > \frac{b}{p_2} \\ \text{any mix on the budget line,} & \frac{a}{p_1} = \frac{b}{p_2} \\ 0, & \frac{a}{p_1} < \frac{b}{p_2} \end{cases}$$

Demand is piecewise-constant with a *kink* where  $a/p_1 = b/p_2$ .

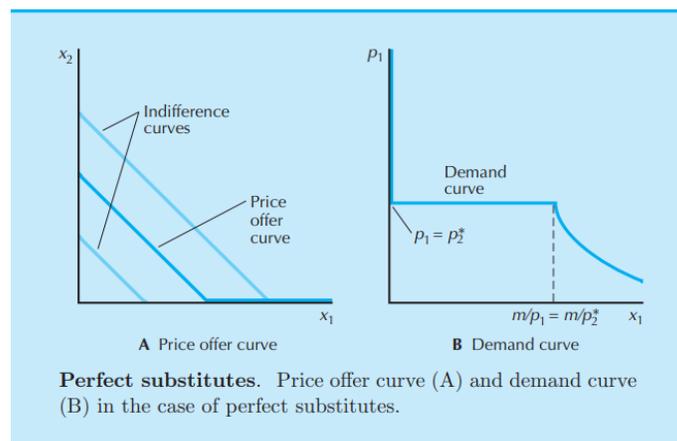


Figure 6.6: Perfect substitutes: price offer and implied demand.

### 6.4.2 Perfect Complements

Let  $U(x_1, x_2) = \min\{ax_1, bx_2\}$  with  $a, b > 0$ .

$$ax_1 = bx_2 \quad \Rightarrow \quad x_2 = \frac{a}{b}x_1.$$

Spend just enough to keep the fixed ratio:

$$x_1^* = \frac{m}{p_1 + \frac{a}{b}p_2}, \quad x_2^* = \frac{a}{b}x_1^*.$$

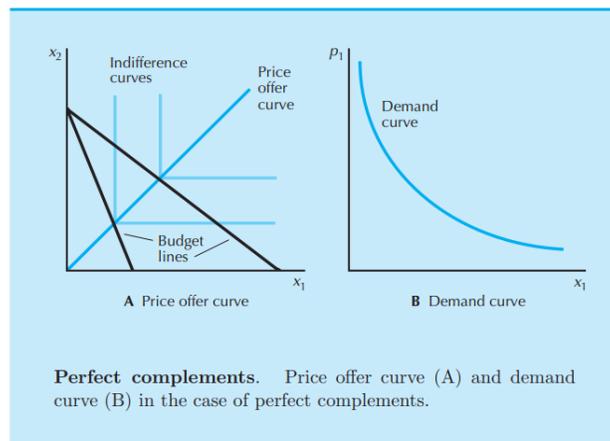


Figure 6.7: Perfect complements: price offer and implied demand.

### 6.4.3 Cobb–Douglas

Let  $U(x_1, x_2) = x_1^\alpha x_2^\beta$  with  $\alpha, \beta > 0$ .

$$x_1^* = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1}, \quad x_2^* = \frac{\beta}{\alpha + \beta} \frac{m}{p_2}.$$

- Demand for each good is linear in  $m$  (homothetic).
- Expenditure shares are constant:  $p_1 x_1^*/m = \alpha/(\alpha + \beta)$ .

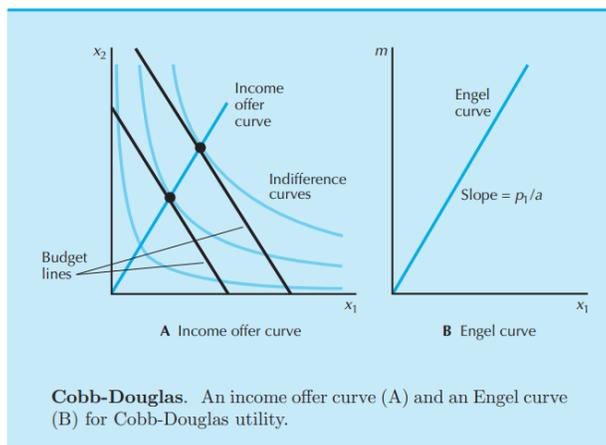


Figure 6.8: Cobb–Douglas: linear Engel curve and straight IOC from the origin.

### 6.4.4 Quasilinear Preferences

Let  $U(x_1, x_2) = v(x_1) + x_2$  (take  $p_2 = 1$  as numeraire).

$$x_2^* = m - p_1 x_1^*, \quad \text{and} \quad v'(x_1^*) = \frac{p_1}{1} \text{ (when interior).}$$

Engel curve for  $x_1$  is *vertical* once the interior solution is reached: additional income all goes to the numeraire.

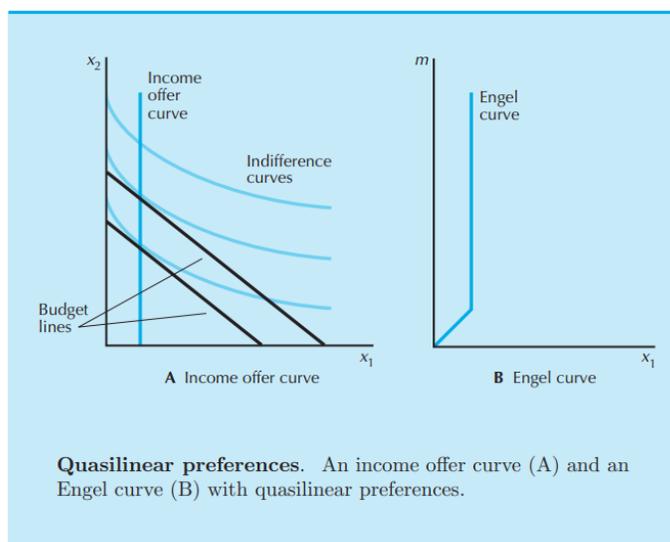


Figure 6.9: Quasilinear: IOC is vertical; Engel curve for  $x_1$  is vertical beyond a threshold.

### 6.4.5 Homothetic Preferences (General)

If preferences are homothetic, scaling income by  $t > 0$  scales demands by  $t$  at fixed prices:

$$x_i(tp_1, tp_2, tm) = t x_i(p_1, p_2, m).$$

Engel curves are straight lines through the origin and IOCs are rays.

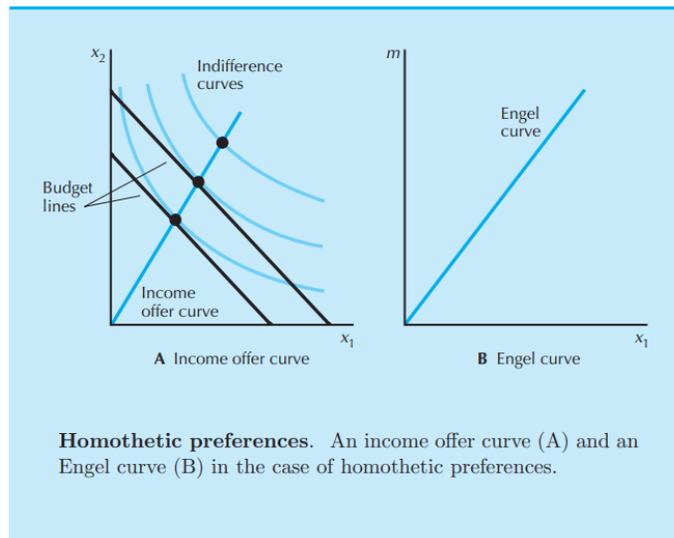


Figure 6.10: Homothetic preferences: straight Engel and ray-like IOC.

## 6.5 Quick Reference: Shapes and Signs

### Key Idea

- **Demand curve** for good 1:  $x_1$  vs.  $p_1$  (others fixed); ordinarily slopes downward. **Inverse demand**:  $p_1(x_1)$ .
- **Income offer curve (IOC)**: optimal bundles as  $m$  varies (prices fixed).
- **Engel curve**:  $m$  vs.  $x_i$ ; upward for normal goods, downward for inferior goods.
- **Perfect substitutes**: corner (all of one good) except at a price tie.
- **Perfect complements**: fixed ratio; demand tracks the diagonal/kink.
- **Cobb–Douglas**: constant budget shares; linear Engel curves.
- **Quasilinear**: income changes go to the numeraire beyond a threshold.
- **Giffen** (rare): demand rises with own price over a range.

## 6.6 (Optional) Elasticity Notation You'll Use Later

Own-price elasticity of demand for good 1:

$$\varepsilon_{x_1, p_1} \equiv \frac{dx_1}{dp_1} \cdot \frac{p_1}{x_1}.$$

Income elasticity:

$$\eta_{x_1, m} \equiv \frac{dx_1}{dm} \cdot \frac{m}{x_1}.$$

Signs: ordinary goods have  $\varepsilon_{x_1, p_1} < 0$ ; normal goods have  $\eta_{x_1, m} > 0$ ; inferior goods have  $\eta_{x_1, m} < 0$ .

## Chapter 7: Consumer's Surplus

### Introduction

Consumer's Surplus measures the difference between the total amount a consumer is willing to pay for a certain quantity of a good and the total amount they actually pay. It provides a practical way to translate satisfaction into monetary terms and to evaluate how much better or worse off consumers are when prices change.

### 7.1 Demand for a Discrete Good

For a discrete good (one that is bought in whole units such as tickets or bottles), each unit has its own **reservation price**, which is the maximum amount the consumer is willing to pay for that specific unit.

- Let  $r_1$  be the reservation price for the first unit,  $r_2$  for the second, and so on, where  $r_1 \geq r_2 \geq r_3 \geq \dots$
- If the market price is  $p$ , the consumer will buy all units for which  $r_i \geq p$ .

**Consumer's Surplus (CS)** is the sum of the differences between each reservation price and the market price for all units purchased:

$$CS = (r_1 - p) + (r_2 - p) + (r_3 - p) + \dots$$

up to the last unit where  $r_i \geq p$ .

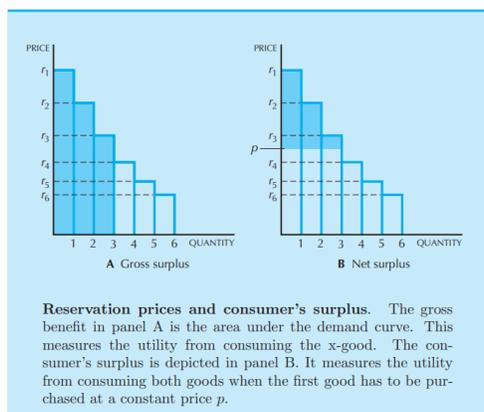


Figure 7.1: Consumer's surplus as the area between reservation prices and the market price for discrete goods.

Each bar in the diagram represents a unit's reservation price. The consumer's surplus is the total area between the tops of the bars (willingness to pay) and the market price line.

## 7.2 Constructing Utility from Demand

The demand function reflects how many units a consumer would choose to buy at each price. Since reservation prices measure willingness to pay for each unit, we can think of them as the “utility values” that generate the demand curve.

Thus, the total utility of consuming  $n$  units can be represented by the sum of the reservation prices:

$$U(n) = r_1 + r_2 + \cdots + r_n.$$

If we subtract what the consumer actually spends ( $p \times n$ ), we get consumer's surplus:

$$CS = U(n) - p \times n.$$

Hence, consumer's surplus measures how much better off the consumer is from buying  $n$  units at price  $p$  instead of not buying the good at all.

## 7.3 Other Interpretations of Consumer's Surplus

Consumer's surplus can also be interpreted as:

- The monetary measure of the **extra benefit** received by consumers in the market.
- The area **under the demand curve and above the price line**.
- A tool to measure welfare changes when prices or income change.

This makes consumer's surplus a convenient way to measure changes in well-being when we cannot directly measure utility.

## 7.4 From Consumer's Surplus to Consumers' Surplus

When we add up the consumer's surplus of all individuals in a market, we obtain the **consumers' surplus** (note the plural), which measures the total benefit to all consumers in the economy.

This concept helps economists compare total welfare under different market conditions—such as price changes, taxes, or price controls.

## 7.5 Approximating a Continuous Demand

For goods that can be bought in continuous quantities, we can approximate the consumer's surplus as the area under the demand curve and above the price line. Even though the demand curve looks continuous, we can think of it as many tiny discrete units stacked together.

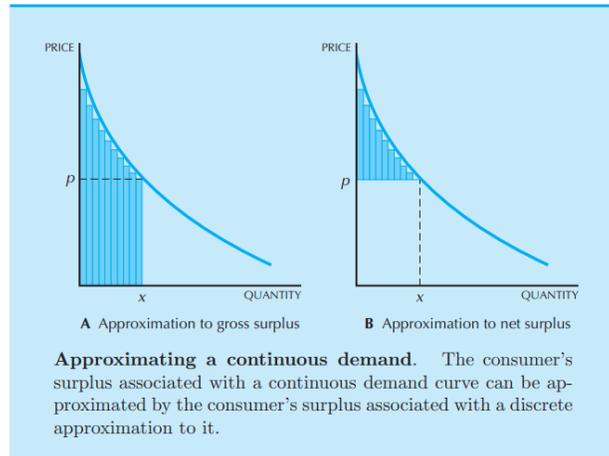


Figure 7.2: Approximating consumer's surplus for a continuous good using small rectangles.

## 7.6 Quasilinear Utility

If a consumer's utility function has the form:

$$U(x, y) = v(x) + y,$$

where  $x$  is the good in question and  $y$  represents money spent on other goods, then the preferences are called **quasilinear**.

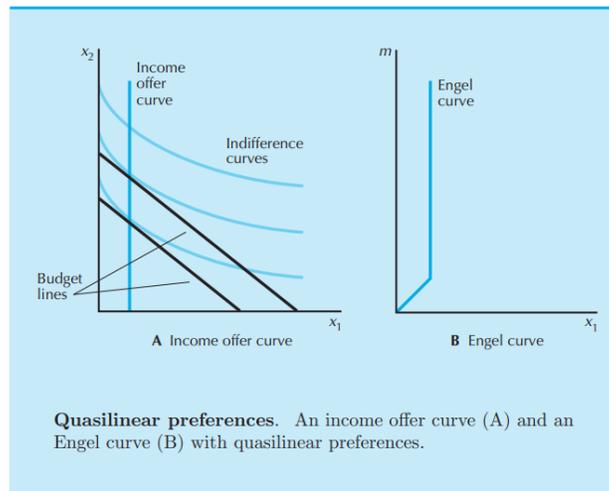


Figure 7.3: Quasilinear preferences: Indifference curves are vertical shifts of one another.

In this case:

- The marginal utility of money is constant.
- Changes in consumer's surplus exactly represent changes in utility measured in monetary terms.

Therefore, with quasilinear utility, consumer's surplus provides an exact measure of welfare change.

## 7.7 Interpreting the Change in Consumer's Surplus

A change in consumer's surplus measures how much better or worse off a consumer is when the price of a good changes.

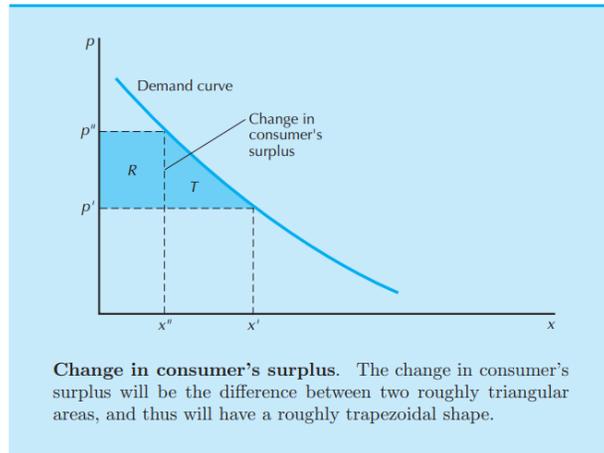


Figure 7.4: Change in consumer's surplus when price changes.

When the price decreases:

- Consumers gain a **rectangle area** representing the savings on units they were already buying.
- They also gain a **triangle area** representing the benefit from new units purchased at the lower price.

When the price increases, the same two areas become losses.

## 7.8 Compensating and Equivalent Variation

Consumer's surplus is an approximate measure of welfare change, but a more precise way is to use **Compensating Variation (CV)** and **Equivalent Variation (EV)**.

- **Compensating Variation (CV):** The amount of money that must be given to (or taken from) the consumer after a price change to restore them to their original level of utility.
- **Equivalent Variation (EV):** The amount of money that would have to be taken from (or given to) the consumer before the price change to make them as well off as after the price change.

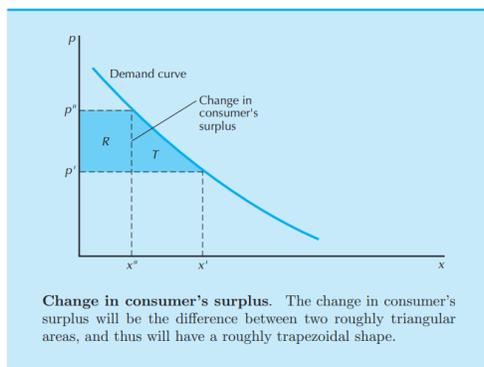


Figure 7.5: Illustrating compensating and equivalent variation around a price change.

Although CV and EV are theoretically superior measures of welfare change, consumer's surplus usually gives a close approximation, especially when income effects are small.

## 7.9 Example: Compensating and Equivalent Variation for Quasilinear Preferences

If preferences are quasilinear (utility  $U(x, y) = v(x) + y$ ), then CV, EV, and the change in consumer's surplus all give the same numerical result.

This is because the marginal utility of income is constant, meaning that the monetary value of a utility change does not depend on how much money the person has.

## 7.10 Producer's Surplus

**Producer's Surplus (PS)** is the difference between what producers actually receive for selling a good and the minimum amount they would be willing to accept.

It measures the net benefit to producers and is shown as the area **above the supply curve and below the market price**.

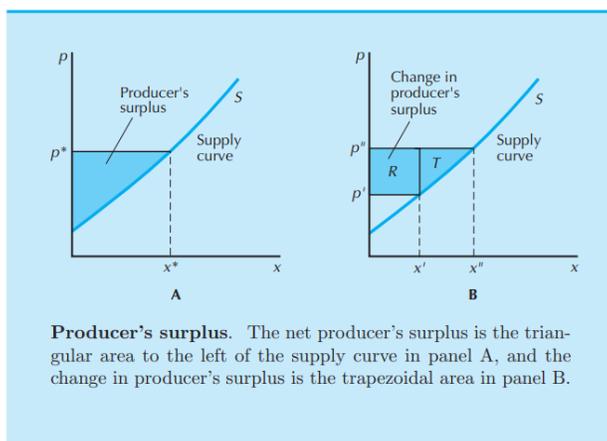


Figure 7.6: Producer's surplus as the area above the supply curve and below the market price.

$$PS = \frac{1}{2} \times (P - P_{\min}) \times Q.$$

## 7.11 Benefit-Cost Analysis

Economists often use changes in consumer's and producer's surplus to evaluate policies and projects.

**Benefit-cost analysis** compares the total gains and losses to all members of society. A project is considered desirable if total benefits exceed total costs—that is, if total surplus increases.

$$\text{Net Benefit} = \Delta CS + \Delta PS.$$

## 7.12 Rationing

Rationing occurs when the government limits the quantity of a good that consumers can purchase, regardless of price.

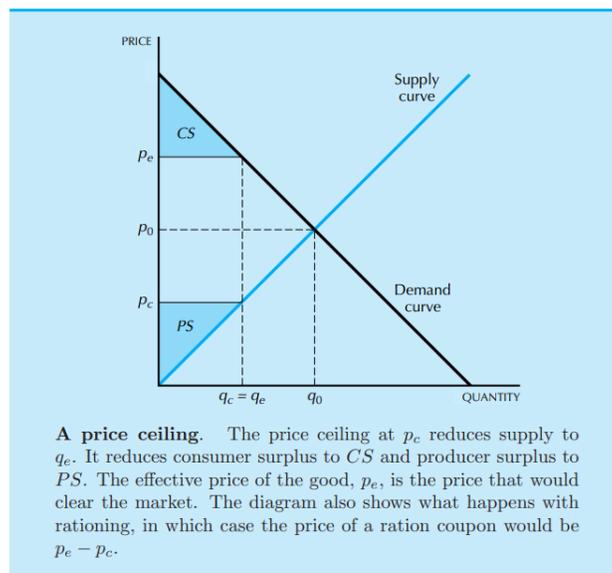


Figure 7.7: Rationing or a price ceiling reduces total surplus and creates a deadweight loss.

This policy prevents some consumers from buying units they value more than it costs to produce, leading to a loss in total surplus and efficiency.

## 7.13 Calculating Gains and Losses

To calculate welfare effects of any market change:

1. Identify the initial and new prices and quantities.

2. Compute the initial and new consumer's and producer's surpluses using geometric areas (triangles or rectangles).
3. Compare the total surpluses before and after the policy change.
4. The difference gives the net gain or deadweight loss.

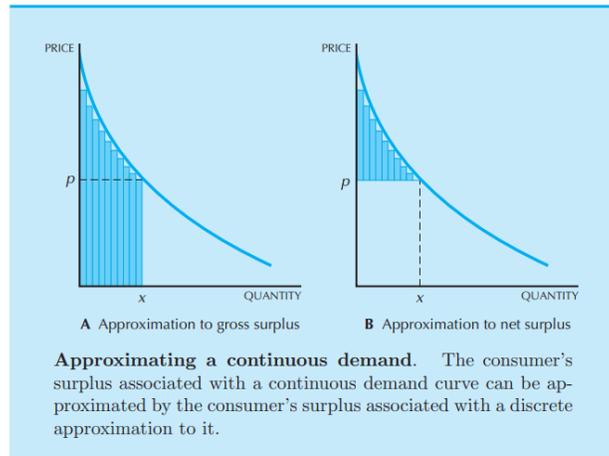


Figure 7.8: Gains and losses from a price change represented by simple geometric areas.

### Example 1: Change in Consumer's Surplus

Demand:  $Q_D = 100 - 2P$ . Initial price  $P_0 = 30 \Rightarrow Q_0 = 40$ . New price  $P_1 = 35 \Rightarrow Q_1 = \boxed{30}$

Consumer's Surplus at each price:

$$CS_0 = \frac{1}{2}(50 - 30)(40) = 400, \quad CS_1 = \frac{1}{2}(50 - 35)(30) = \boxed{225}$$

Change in consumer's surplus:

$$\Delta CS = CS_1 - CS_0 = \boxed{-175}$$

So, consumers lose \$175 when price rises from 30 to 35.

### Example 2: Price Ceiling and Welfare

Demand:  $Q_D = 90 - P$ . Supply:  $Q_S = 2P - 60$ .

Equilibrium:  $P^* = 50, Q^* = 40$ .

$$CS^* = \frac{1}{2}(90 - 50)(40) = 800, \quad PS^* = \frac{1}{2}(50 - 30)(40) = \boxed{400}$$

With a price ceiling  $P_c = 40$ :

$$Q_D = 50, \quad Q_S = 20, \quad Q_c = 20.$$

$$CS_c = \frac{1}{2}(90 - 40)(20) = 500, \quad PS_c = \frac{1}{2}(40 - 30)(20) = \boxed{100}$$

$$TS_c = 500 + 100 = 600, \quad DWL = 1200 - 600 = \boxed{600}$$

Hence, total surplus falls by \$600

## Chapter 8: Market Demand

### Introduction

Market demand shows how the total quantity demanded of a good changes as its price changes, aggregating all consumers' individual demands. Here, we look at how individual demand curves combine into a market demand curve, how to measure responsiveness using elasticity, and how these concepts determine total and marginal revenue.

### 8.1 From Individual to Market Demand

Each individual has their own demand function, showing the quantity of a good they want at each price. The market demand curve is the **horizontal sum** of all individual demand curves.

$$Q^M(p) = Q_1(p) + Q_2(p) + Q_3(p) + \dots$$

That is, for each price level, we add up the quantities that all consumers demand.

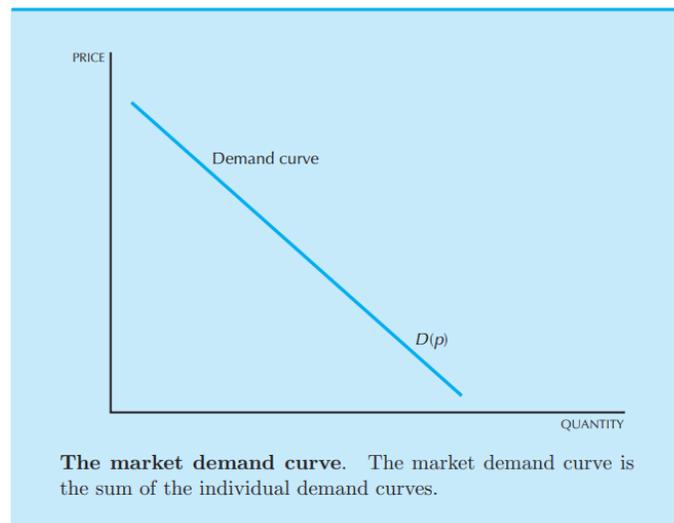


Figure 8.1: The market demand curve is the horizontal sum of individual demand curves.

## 8.2 The Inverse Demand Function

Instead of expressing quantity as a function of price ( $Q = D(p)$ ), we can express price as a function of quantity:

$$p = P(Q).$$

This is called the **inverse demand function**. It tells us the maximum price at which a given quantity of the good can be sold in the market.

- The inverse demand curve slopes downward.
- It is useful for computing total revenue and marginal revenue later.

## 8.3 Example: Adding Up Linear Demand Curves

When we have multiple consumers with linear demand functions, the market demand curve is their horizontal sum.

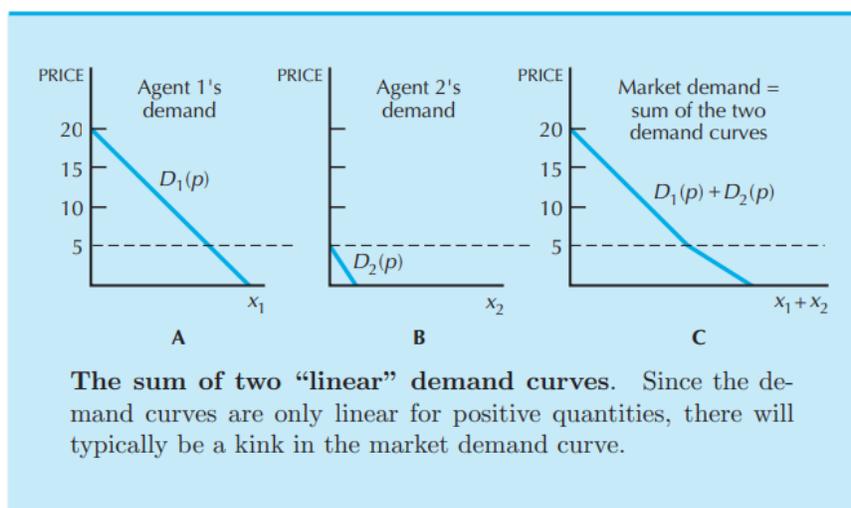


Figure 8.2: Summing two individual linear demand curves to form a market demand curve.

### Example:

$$D_1(p) = 20 - p, \quad D_2(p) = 15 - p.$$

The market demand curve is:

$$D_M(p) = D_1(p) + D_2(p) = 35 - 2p.$$

Notice the kink that occurs if one consumer stops buying earlier than the other, demand curves are only linear for positive quantities.

## 8.4 Discrete Goods

If a good can only be purchased in whole units, the market demand is obtained by summing individual step functions across consumers.

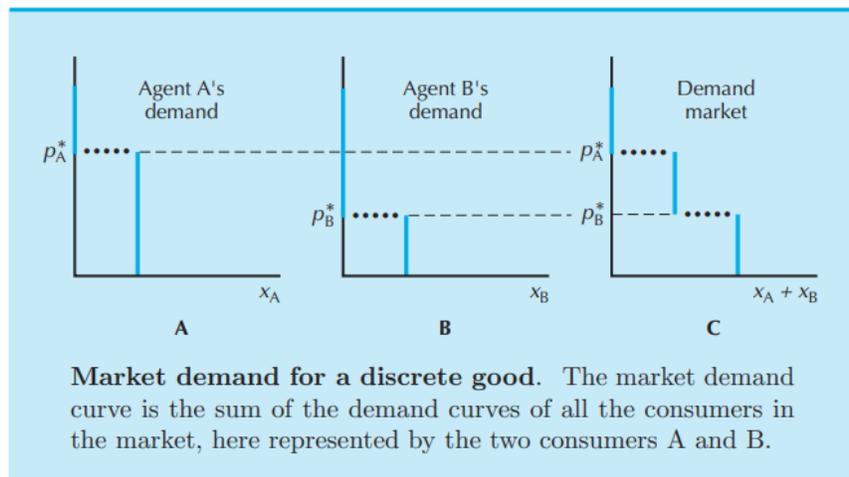


Figure 8.3: Market demand for a discrete good is the sum of individual step functions.

Each step in the market demand represents another consumer entering the market as price falls below their reservation price.

## 8.5 The Extensive and the Intensive Margin

The total market demand can change in two ways:

- **Extensive margin:** more consumers decide to buy the good (number of buyers changes).
- **Intensive margin:** existing consumers buy more or less of the good (quantity per buyer changes).

These two effects together explain how total demand responds to changes in price.

## 8.6 Elasticity of Demand

The **price elasticity of demand** measures how responsive quantity demanded is to price changes.

$$\varepsilon = \frac{dq}{dp} \cdot \frac{p}{q}.$$

### Interpretation:

- $\varepsilon < -1$ : demand is **elastic** —> consumers are highly responsive to price.
- $\varepsilon = -1$ : demand is **unit elastic**.
- $\varepsilon > -1$ : demand is **inelastic** —> consumers respond weakly to price changes.

Elasticity depends on both the slope of the demand curve and the price–quantity combination.

## 8.7 Example: The Elasticity of a Linear Demand Curve

Consider a linear demand function:

$$p = a - bq.$$

**Elasticity:**

$$\varepsilon = \frac{dp}{dq} \cdot \frac{q}{p} = (-b) \frac{q}{a - bq}.$$

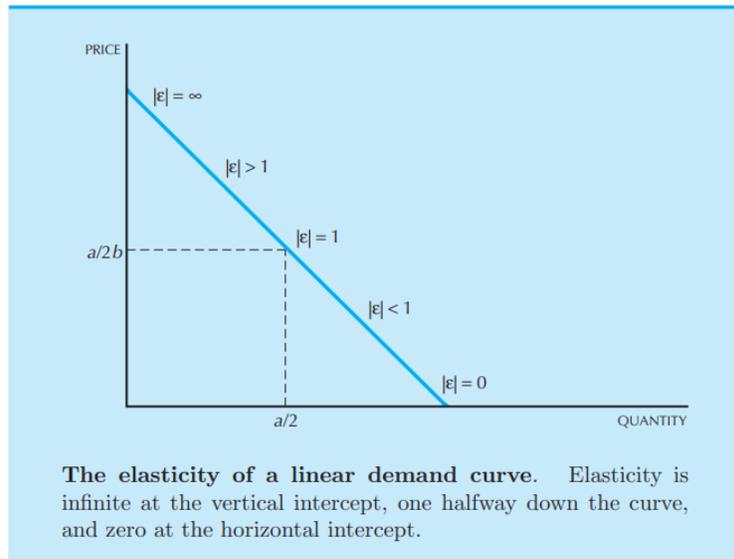


Figure 8.4: Elasticity of a linear demand curve.

- Elasticity is infinite at the vertical intercept ( $q = 0$ ).
- Elasticity equals  $-1$  halfway down the curve.
- Elasticity is zero at the horizontal intercept ( $p = 0$ ).

Hence, a linear demand curve is elastic at high prices, unit elastic at midpoint, and inelastic at low prices.

## 8.8 Elasticity and Demand

Elasticity helps describe how total expenditure or total revenue changes with price.

**Total Revenue (TR):**

$$R = p \cdot q.$$

**Change in Revenue:** A price increase affects revenue in two opposite ways:

1. It increases revenue from existing units (price effect).
2. It decreases revenue from lost units (quantity effect).

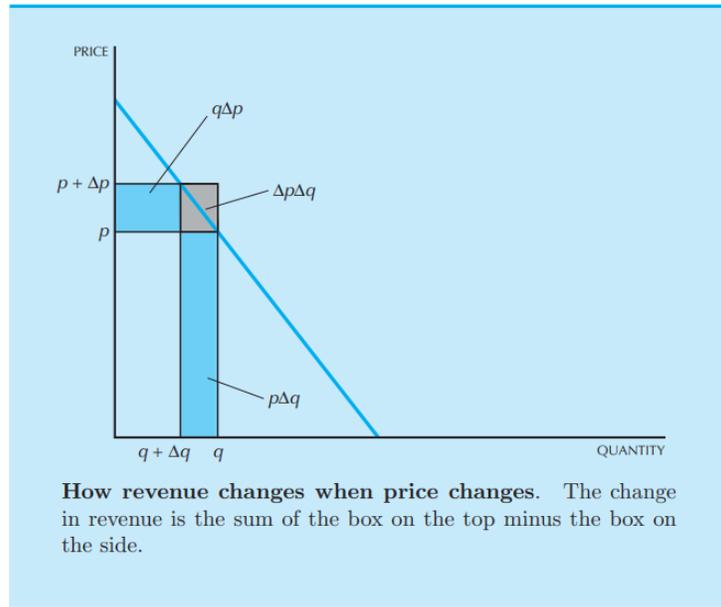


Figure 8.5: How total revenue changes when price changes.

**Elasticity and Revenue:**

$$\frac{dR}{dp} = q + p \frac{dq}{dp} = q \left( 1 + \frac{1}{\varepsilon} \right).$$

- If demand is elastic ( $|\varepsilon| > 1$ ),  $dR/dp < 0$ : increasing price reduces revenue.
- If demand is inelastic ( $|\varepsilon| < 1$ ),  $dR/dp > 0$ : increasing price raises revenue.
- If demand is unit elastic ( $|\varepsilon| = 1$ ),  $dR/dp = 0$ : revenue is maximized.

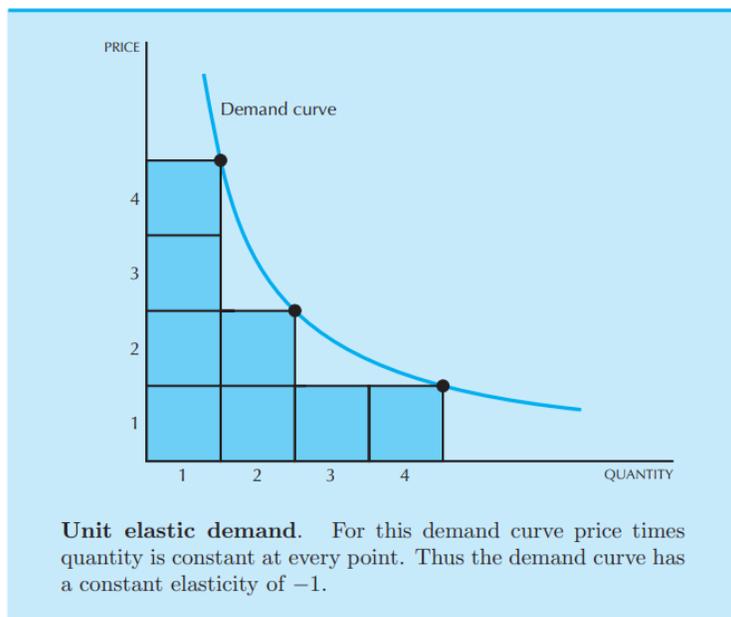


Figure 8.6: Unit elastic demand: total expenditure ( $p \times q$ ) is constant at all points.

### 8.9 Example: Strikes and Profits

Suppose demand for steel is inelastic in the short run. If a strike reduces output, the total quantity falls, but price rises sharply. Because the price increase outweighs the quantity decrease, total revenue and producers' profits may actually rise, even though output fell.

This example shows how elasticity determines whether a price increase raises or lowers total revenue.

### 8.10 Constant Elasticity Demands

For a demand curve of the form

$$q = Ap^{-\varepsilon},$$

elasticity is constant and equal to  $-\varepsilon$  at every point.

$$\varepsilon = -\frac{dq}{dp} \cdot \frac{p}{q} = -\varepsilon.$$

**Inverse demand function:**

$$p = Bq^{-1/\varepsilon}.$$

Constant elasticity demand curves are commonly used because they simplify analysis of revenue and marginal revenue.

### 8.11 Elasticity and Marginal Revenue

Marginal revenue (MR) is the additional revenue from selling one more unit.

$$MR = \frac{dR}{dq} = p + q \frac{dp}{dq}.$$

Substitute  $\frac{dp}{dq}$  from the elasticity definition:

$$MR = p \left( 1 + \frac{1}{\varepsilon} \right).$$

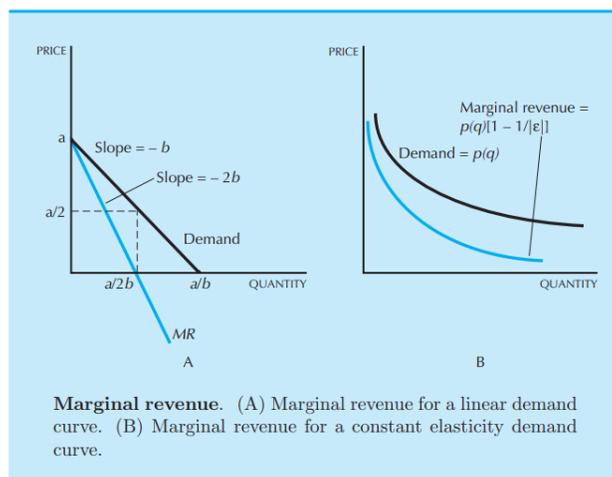


Figure 8.7: Marginal revenue for a linear and a constant-elasticity demand curve.

- If demand is elastic ( $|\varepsilon| > 1$ ),  $MR > 0$ .
- If demand is inelastic ( $|\varepsilon| < 1$ ),  $MR < 0$ .
- When demand is unit elastic ( $|\varepsilon| = 1$ ),  $MR = 0$ .

### 8.12 Example: Setting a Price

A firm facing demand  $p = 100 - 2q$  has:

$$R = pq = 100q - 2q^2, \quad MR = \frac{dR}{dq} = 100 - 4q.$$

If marginal cost (MC) = 20, set  $MR = MC$ :

$$100 - 4q = 20 \Rightarrow q = 20, \quad p = 100 - 2(20) = 60.$$

Profit-maximizing price is  $p = 60$ , quantity  $q = 20$ .

Elasticity at this point:

$$\varepsilon = \frac{dp}{dq} \cdot \frac{q}{p} = (-2) \frac{20}{60} = -\frac{2}{3}.$$

Since  $|\varepsilon| < 1$ , demand is inelastic, and price is above the midpoint—consistent with  $MR < 0$ .

### 8.13 Marginal Revenue Curves

For a linear demand  $p = a - bq$ , total revenue is:

$$R = aq - bq^2,$$

and marginal revenue is:

$$MR = a - 2bq.$$

Thus, the MR curve has the same intercept but twice the slope of the demand curve.

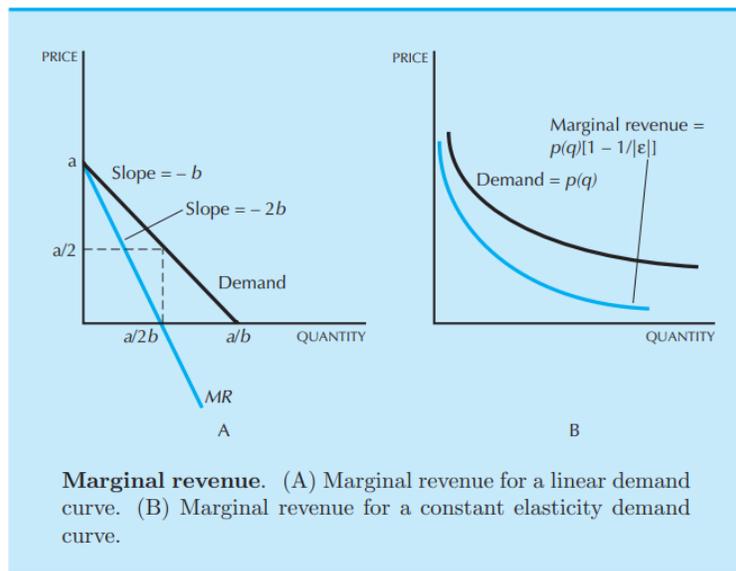


Figure 8.8: Marginal revenue curve lies below demand and has twice the slope.

### 8.14 Income Elasticity of Demand

The **income elasticity of demand** measures how quantity demanded changes with income:

$$\eta = \frac{dq}{dm} \cdot \frac{m}{q}.$$

- $\eta > 0$ : **Normal good**  $\rightarrow$  demand increases with income.
- $\eta < 0$ : **Inferior good**  $\rightarrow$  demand decreases with income.

Income elasticity helps classify goods and predict how changes in income affect market demand as a whole.

## Chapter 9: Equilibrium

### Introduction

We study how demand and supply interact to determine prices and quantities in a competitive market. The point where the market clears, where quantity demanded equals quantity supplied, is known as the **equilibrium**. At equilibrium, no buyer or seller has an incentive to change their behavior, and the outcome is considered **Pareto efficient**, meaning no one can be made better off without making someone else worse off.

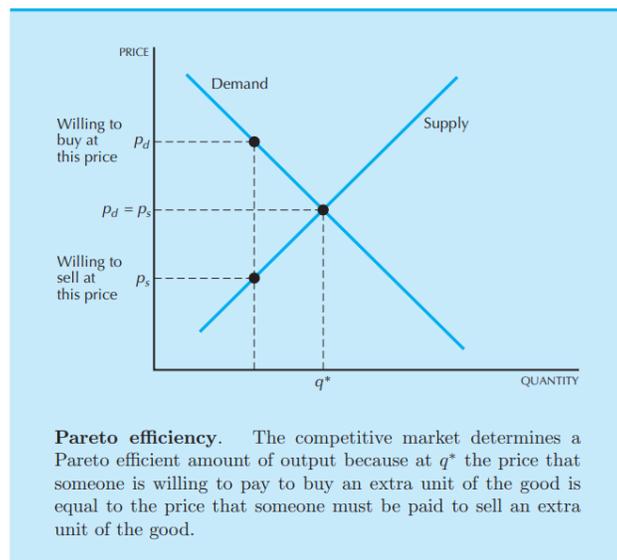


Figure 9.1: Pareto efficiency. At  $q^*$ , willingness to pay equals willingness to sell, ensuring a Pareto efficient outcome.

### 9.1 Supply and Market Equilibrium

The **demand curve** represents consumers' willingness to pay for each unit, while the **supply curve** represents producers' willingness to sell. In a competitive market:

$$Q_D(p) = Q_S(p)$$

determines the equilibrium price  $p^*$  and quantity  $q^*$ .

**Example (Linear Case):** Let

$$p_D(q) = a - bq \quad \text{and} \quad p_S(q) = c + dq$$

Setting  $p_D(q^*) = p_S(q^*)$  gives:

$$q^* = \frac{a - c}{b + d}, \quad p^* = a - bq^* = \frac{ad + bc}{b + d}$$

The equilibrium occurs where the two curves intersect.

## 9.2 Two Special Cases

- **Vertical supply curve (perfectly inelastic):** Quantity is fixed; price is determined entirely by demand.
- **Horizontal supply curve (perfectly elastic):** Price is fixed; quantity adjusts to demand.

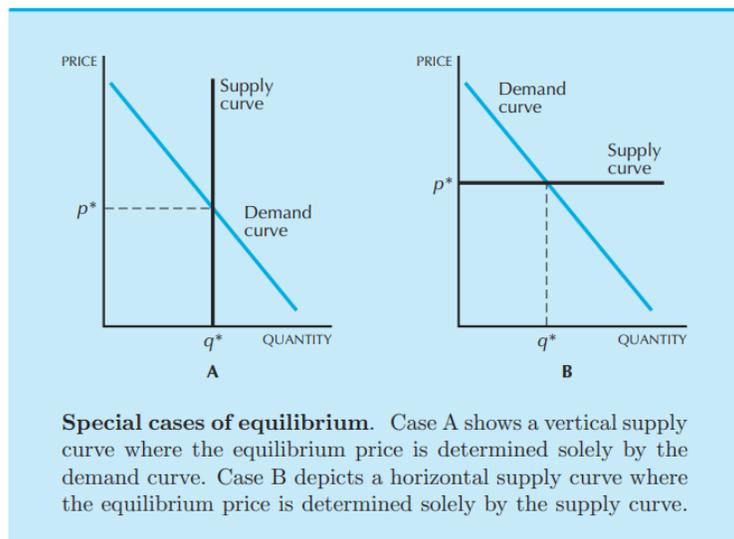


Figure 9.2: Special cases of equilibrium. Case A shows vertical supply; Case B shows horizontal supply.

## 9.3 Inverse Demand and Supply Functions

Inverse functions express price as a function of quantity:

$$p_D(q) = a - bq, \quad p_S(q) = c + dq.$$

These are particularly useful for calculating equilibrium and for finding the effects of taxes or subsidies.

## 9.4 Example: Equilibrium with Linear Curves

Let  $p_D(q) = 120 - 2q$  and  $p_S(q) = 40 + q$ . Equating:

$$120 - 2q = 40 + q \Rightarrow q^* = 26.67, \quad p^* = 66.67.$$

Thus, equilibrium price is \$66.67 and equilibrium quantity is 26.67 units.

## 9.5 Comparative Statics

**Comparative statics** analyzes how equilibrium changes when parameters shift.

$$\frac{\partial q^*}{\partial a} = \frac{1}{b+d} > 0, \quad \frac{\partial q^*}{\partial c} = -\frac{1}{b+d} < 0$$

This means:

- An increase in demand ( $a \uparrow$ ) raises both  $p^*$  and  $q^*$
- An increase in supply ( $c \downarrow$ ) raises  $q^*$  and lowers  $p^*$

### Intuition Behind Comparative Statics

**Comparative statics** shows how equilibrium price and quantity change when market conditions, such as demand, supply, taxes, or income—shift. It compares two equilibria: one before the change and one after, focusing on the *direction and size* of the adjustment rather than the process itself.

**Examples:** An increase in income shifts demand rightward  $\Rightarrow$  both price and quantity rise. Higher production costs shift supply leftward  $\Rightarrow$  price rises and quantity falls.

The magnitude of these changes depends on elasticity: flatter curves (more elastic) cause larger quantity adjustments and smaller price changes.

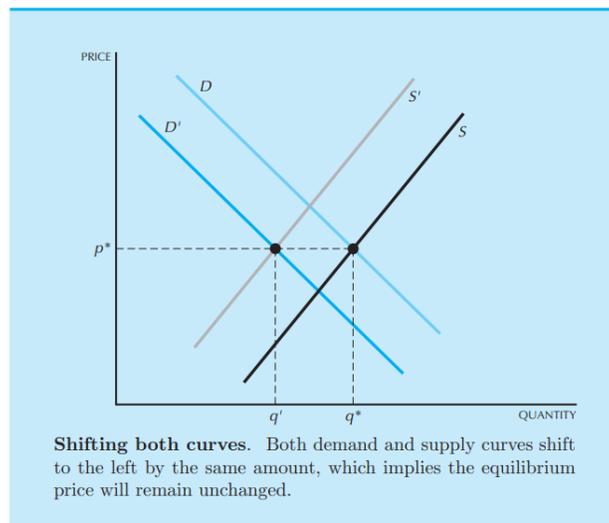


Figure 9.3: Shifting both curves. When both demand and supply shift left equally,  $p^*$  remains unchanged.

## 9.6 Taxes

A **specific tax**  $t$  per unit can be represented as a vertical shift between the price paid by buyers ( $p_d$ ) and the price received by sellers ( $p_s$ ):

$$p_d = p_s + t$$

It can be modeled as either:

- a downward shift of the demand curve by  $t$ , or
- an upward shift of the supply curve by  $t$ .

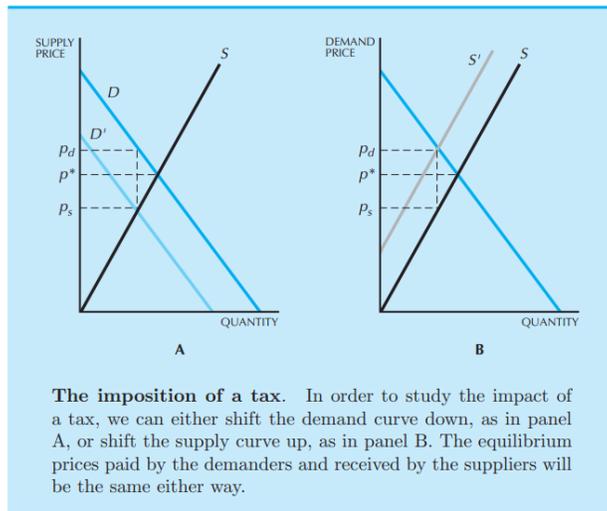


Figure 9.4: The imposition of a tax. It can be shown as a shift in demand or supply; both yield the same equilibrium.

### 9.7 Example: Taxation with Linear Demand and Supply

Let  $p_D(q) = 100 - 2q$  and  $p_S(q) = 20 + q$ . With a tax  $t = 10$ :

$$100 - 2q = 20 + q + 10 \Rightarrow q_t = 23.3, \quad p_d = 100 - 2(23.3) = 53.3, \quad p_s = p_d - 10 = 43.3$$

Tax revenue =  $t \times q_t = 10 \times 23.3 = 233$

The **incidence of tax** (who bears the burden) depends on elasticities:

$$\text{Buyers' share} = \frac{d}{b + d}, \quad \text{Sellers' share} = \frac{b}{b + d}.$$

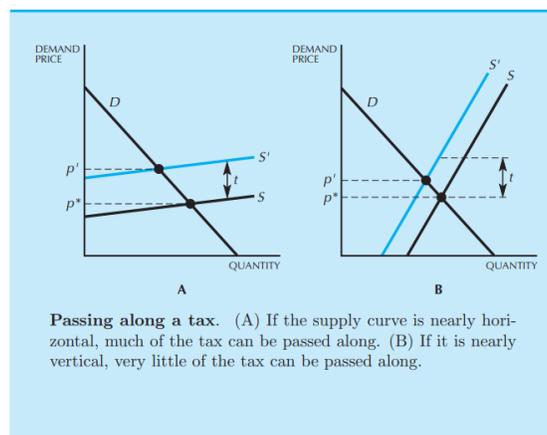


Figure 9.5: Passing along a tax. The side of the market that is less elastic bears more of the burden.

## 9.8 The Deadweight Loss of a Tax

The reduction in total surplus caused by a tax is the **deadweight loss (DWL)**.

$$DWL = \frac{1}{2} t (q^* - q_t).$$

The greater the elasticities of demand and supply, the larger the deadweight loss.

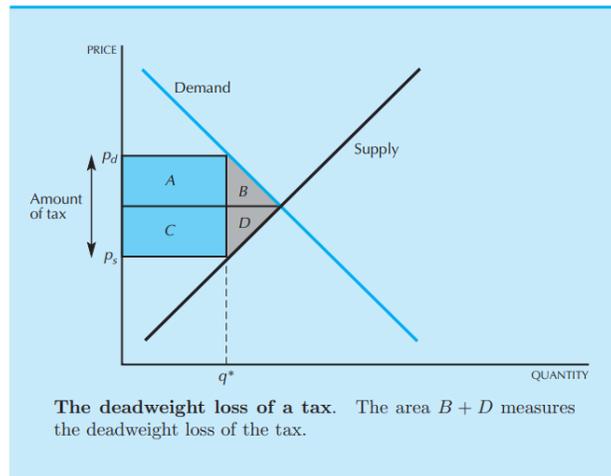


Figure 9.6: Deadweight loss of a tax. The shaded triangle ( $B + D$ ) represents efficiency loss.

## 9.9 Example: The Market for Loans

In the loanable funds market:

- **Supply** represents savings behavior.
- **Demand** represents borrowers' willingness to invest.

An interest income tax shifts the supply of funds leftward (savers require higher gross interest rates). This raises the borrowing rate, lowers the saving rate, and reduces total lending—creating a deadweight loss similar to that of a goods tax.

## 9.10 Example: Food Subsidies

A **subsidy** is the opposite of a tax, it reduces buyers' prices or increases sellers' receipts. If the government pays  $s$  per unit sold, then:

$$p_s = p_d + s.$$

This increases quantity traded but introduces inefficiency because more is produced than the efficient level. The deadweight loss area measures the excess cost of producing subsidized units.

### 9.11 Example: Subsidies in Iraq

In Iraq, government subsidies on fuel and food have historically kept prices below market levels. While this helped consumers in the short run, it also led to:

- Overconsumption and resource misallocation.
- Large fiscal costs to maintain subsidies.
- Black-market reselling and inefficiency.

Economically, broad subsidies expand consumption beyond the efficient equilibrium quantity and create welfare losses similar in shape to a tax, but on the opposite side.

### 9.12 Pareto Efficiency

The competitive equilibrium quantity  $q^*$  is Pareto efficient because:

- The willingness to pay for the last unit equals the willingness to accept.
- There are no mutually beneficial trades left unexploited.

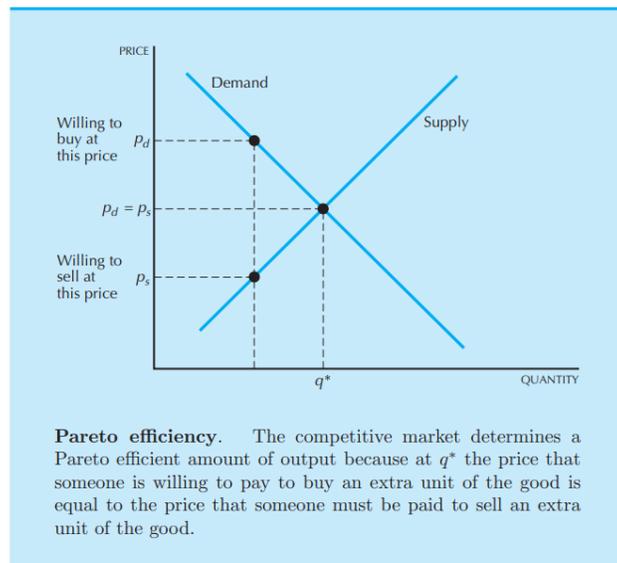


Figure 9.7: Pareto efficiency at competitive equilibrium.

#### Takeaways:

- Market equilibrium occurs when  $Q_D = Q_S$ , where no one wants to change their decisions.
- Linear inverse demand and supply functions simplify solving for equilibrium and comparative statics.
- Specific taxes raise consumer prices and lower producer prices, causing inefficiency and deadweight loss.
- The less elastic side of the market bears more of the tax burden.
- Subsidies increase production beyond the efficient level and can also generate deadweight loss.
- The competitive equilibrium quantity is Pareto efficient, mutually beneficial trades are exhausted.

## Chapter 10: Exchange

### Big Picture

Here we study how two consumers can trade goods with each other and how prices coordinate these trades. The central tools are the **Edgeworth box**, **Pareto efficiency**, and **competitive equilibrium**. The key question is simple: when is an allocation efficient, and when can voluntary trade make people better off?

#### Key Idea

An exchange economy is efficient if there are no mutually beneficial trades left. Prices matter because they guide individuals toward such efficient outcomes without central coordination.

### 10.1 The Edgeworth Box

The **Edgeworth box** is a graphical device that represents all possible allocations of two goods between two people.

- Width of the box = total amount of good 1.
- Height of the box = total amount of good 2.
- Person A's origin is at the lower-left corner.
- Person B's origin is at the upper-right corner.

Every point in the box represents a feasible allocation: what A gets automatically determines what B gets.

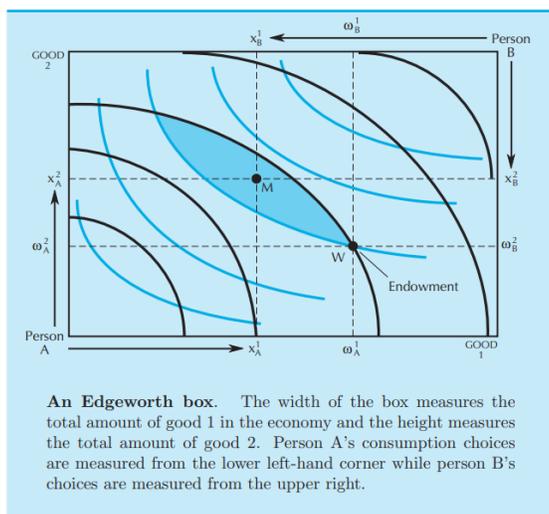


Figure 10.1: The Edgeworth box. Any point represents a feasible allocation of goods between two people.

**Key Idea**

Feasibility is automatic in the Edgeworth box. What matters is whether an allocation is desirable or efficient.

**10.2 Endowments and Trade**

An **endowment** is the initial allocation of goods before trade takes place. It is represented by a point  $W$  inside the Edgeworth box.

From the endowment, individuals can trade voluntarily if both can move to a higher indifference curve.

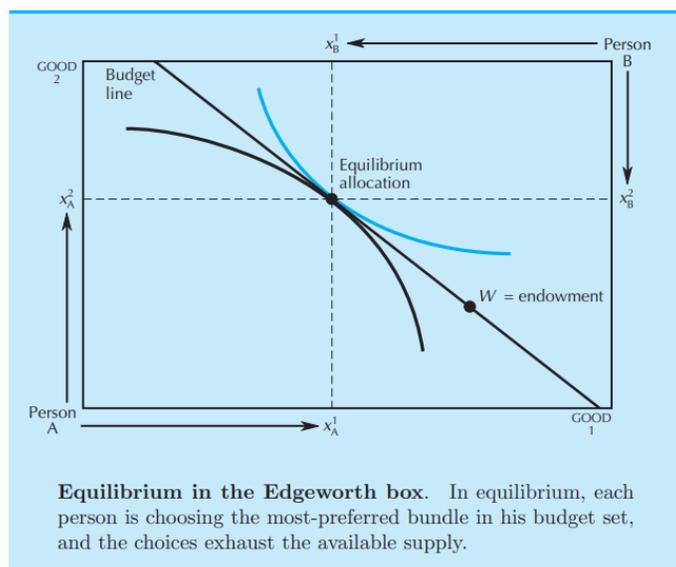


Figure 10.2: Endowment and equilibrium in the Edgeworth box. Trade reallocates goods while preserving feasibility.

Trade is:

- **Voluntary:** both parties must agree.
- **Mutually beneficial:** both end up on higher indifference curves.

**10.3 Pareto Improvement and Pareto Efficiency**

A **Pareto improvement** is a change that makes at least one person better off without making anyone worse off.

An allocation is **Pareto efficient** if no Pareto improvements are possible.

**Key Idea**

At a Pareto efficient allocation, the marginal rates of substitution (MRS) of the two consumers are equal:

$$MRS_A = MRS_B.$$

If they differ, there exists a mutually beneficial trade.

## 10.4 The Contract Curve

The **contract curve** is the set of all Pareto efficient allocations inside the Edgeworth box.

- At every point on the contract curve, the two indifference curves are tangent.
- No further gains from trade are possible.

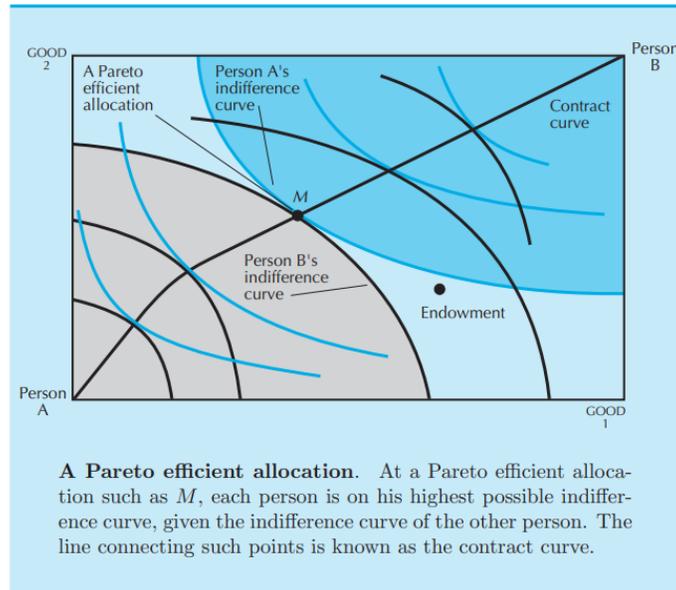


Figure 10.3: The contract curve: the set of Pareto efficient allocations.

Every competitive equilibrium must lie on the contract curve, but not every point on the contract curve is a competitive equilibrium.

## 10.5 Competitive Equilibrium in Exchange

A **competitive equilibrium** consists of:

- Prices  $(p_1, p_2)$
- An allocation where each person maximizes utility subject to their budget constraint
- Markets clearing: total demand equals total supply

Each consumer solves:

$$\max u_i(x_1^i, x_2^i) \quad \text{s.t.} \quad p_1 x_1^i + p_2 x_2^i = p_1 \omega_1^i + p_2 \omega_2^i$$

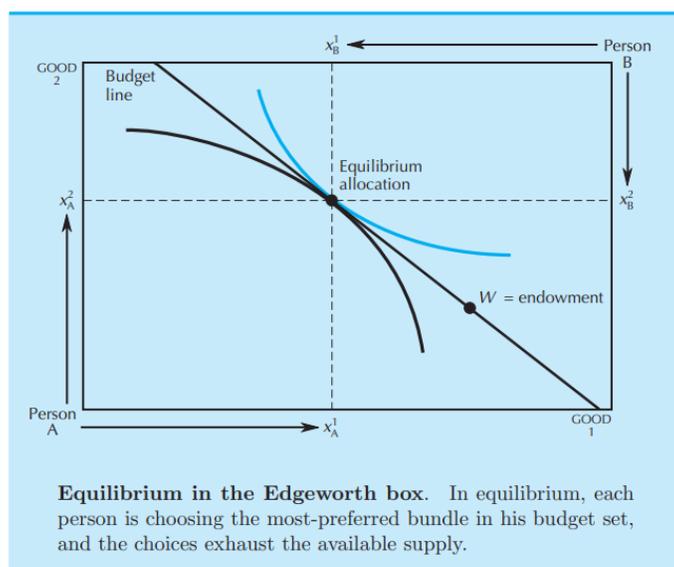


Figure 10.4: Competitive equilibrium in the Edgeworth box. Budget lines support a Pareto efficient allocation.

### Key Idea

At competitive equilibrium:

$$MRS_A = MRS_B = \frac{p_1}{p_2}.$$

Prices align private incentives with efficiency.

## 10.6 First Welfare Theorem

### Key Idea

**First Welfare Theorem:** Every competitive equilibrium allocation is Pareto efficient.

This result holds under:

- Perfect competition
- Complete markets
- No externalities
- Well-behaved preferences

It does **not** say that competitive equilibria are fair, only that they are efficient.

## 10.7 Monopoly in the Edgeworth Box

If one agent has market power, they may manipulate prices or quantities to their advantage.

In a **monopoly exchange**, one person chooses a point on the other person's offer curve that maximizes their own utility.

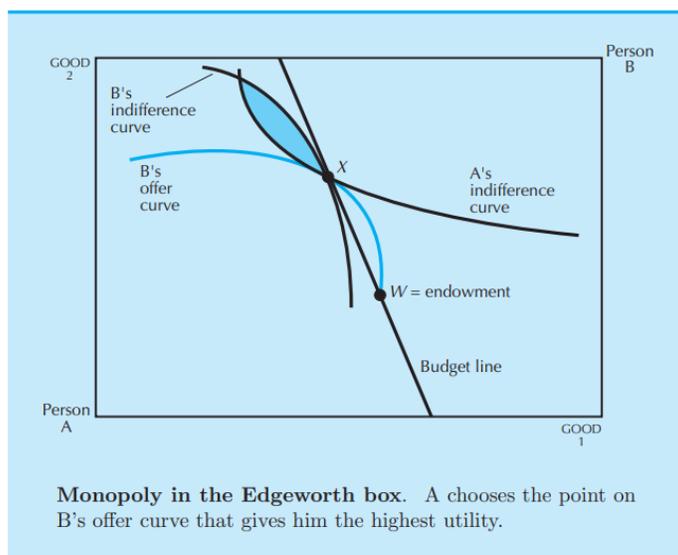


Figure 10.5: Monopoly in the Edgeworth box. The monopolist selects the most favorable point on the opponent's offer curve.

### Key Idea

Monopoly outcomes are generally **Pareto inefficient**. The monopolist restricts trade to extract surplus, leaving mutually beneficial trades unrealized.

## 10.8 Offer Curves

An **offer curve** shows how a consumer's optimal consumption changes as relative prices change, holding endowment fixed.

- Derived from utility maximization
- Lives inside the Edgeworth box
- Competitive equilibrium occurs where offer curves intersect

Offer curves summarize all possible net trades a consumer is willing to make at different prices.

## 10.9 Second Welfare Theorem

### Key Idea

**Second Welfare Theorem:** Any Pareto efficient allocation can be achieved as a competitive equilibrium for some redistribution of endowments.

Interpretation:

- Separate **equity** from **efficiency**.
- First redistribute endowments (lump-sum).
- Then let markets work.

# Chapter 11: Technology

## 11.1 Introduction and Purpose

Technology describes the set of all feasible ways a firm can transform inputs into output. Understanding technology is essential because all cost and supply behavior ultimately comes from the production side of the firm.

## 11.2 The Production Set

The **production set** describes all technologically feasible combinations of inputs and outputs.

Let  $x = (x_1, x_2)$  be inputs and  $y$  be output. The production set  $Y$  consists of all feasible  $(x_1, x_2, y)$  such that output  $y$  can be produced using inputs  $(x_1, x_2)$ .

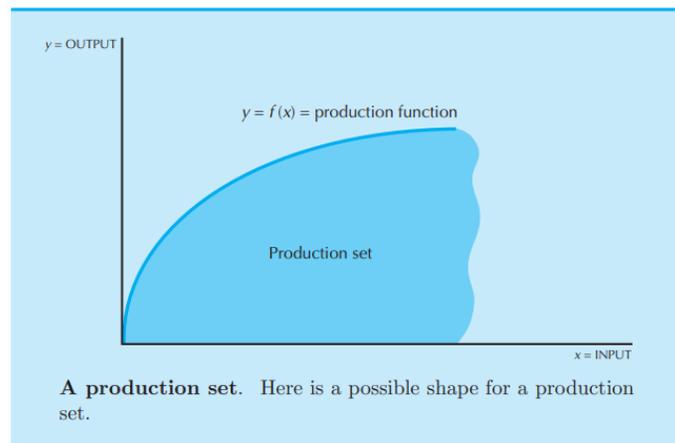


Figure 11.1: A production set

### Key properties of a production set:

- **Free disposal:** If  $(x, y)$  is feasible, then producing less output with the same inputs is also feasible.
- **Non-negativity:** Inputs and outputs are typically assumed to be non-negative.

The upper boundary of the production set corresponds to the maximum output achievable for given inputs.

### 11.3 Production Functions

A **production function** summarizes the technology by giving the maximum output achievable from a given input bundle:

$$y = f(x_1, x_2).$$

The production function represents the *frontier* of the production set.

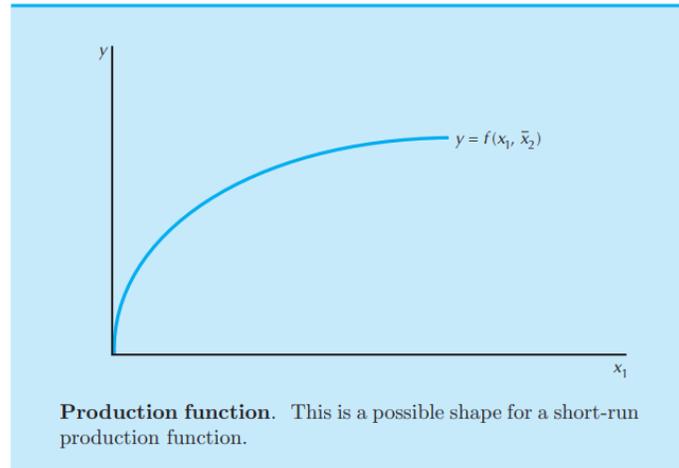


Figure 11.2: A short-run production function

**Interpretation:**

- For any input level, the production function gives the highest feasible output.
- Points below the curve are feasible but inefficient.
- Points above the curve are infeasible.

### 11.4 Short Run vs Long Run

**Short run:** At least one input is fixed. **Long run:** All inputs are variable.

In the short run, production functions often exhibit diminishing marginal returns due to fixed inputs.

### 11.5 Marginal Products

The **marginal product** of input  $x_i$  measures the additional output obtained from increasing that input slightly, holding other inputs constant:

$$MP_i = \frac{\partial f(x_1, x_2)}{\partial x_i}.$$

**Economic intuition:**

- Marginal products capture productivity at the margin.
- Diminishing marginal products are common due to congestion or fixed inputs.

## 11.6 Isoquants

An **isoquant** shows all combinations of inputs that produce the same level of output.

- Analogous to indifference curves in consumer theory.
- Downward sloping due to substitutability.
- Further from the origin means higher output.

### 11.6.1 Convexity of Isoquants

If production plans can be mixed, isoquants are convex.

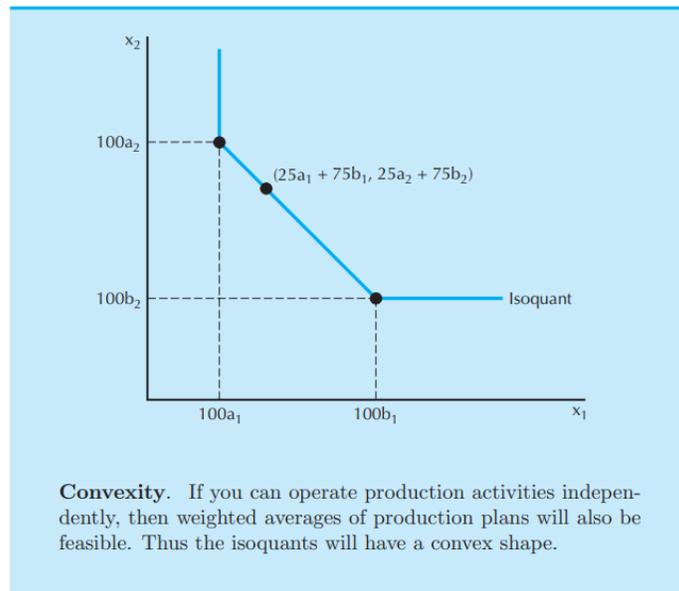


Figure 11.3: Convexity of isoquants

Convexity reflects a diminishing marginal rate of technical substitution.

## 11.7 Marginal Rate of Technical Substitution

The **Marginal Rate of Technical Substitution (MRTS)** measures how much of input  $x_2$  can be reduced when input  $x_1$  is increased by one unit while keeping output constant:

$$MRTS_{12} = -\frac{dx_2}{dx_1} = \frac{MP_1}{MP_2}.$$

**Interpretation:**

- MRTS reflects the tradeoff between inputs.
- A diminishing MRTS corresponds to convex isoquants.

## 11.8 Special Technologies

### 11.8.1 Perfect Substitutes

Inputs are perfect substitutes if they can replace each other at a constant rate:

$$f(x_1, x_2) = x_1 + x_2.$$

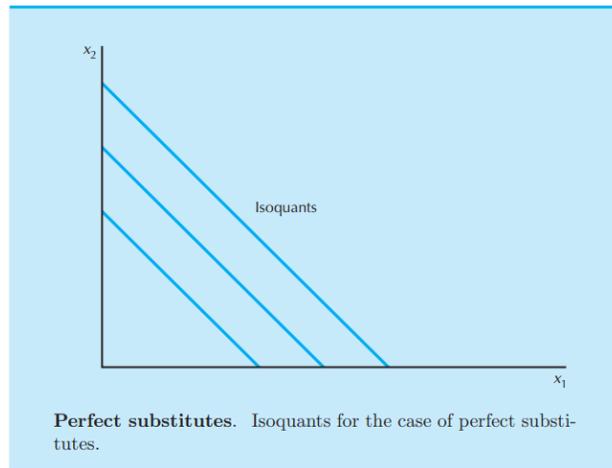


Figure 11.4: Isoquants with perfect substitutes

Isoquants are straight lines with constant slope.

### 11.8.2 Fixed Proportions (Perfect Complements)

Inputs must be used in fixed ratios:

$$f(x_1, x_2) = \min\{x_1, x_2\}.$$

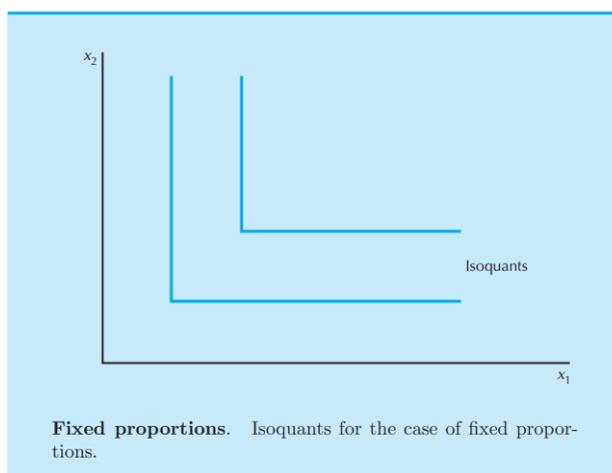


Figure 11.5: Isoquants with fixed proportions

**Key insight:** No substitution is possible. Extra units of one input alone do not increase output.

## 11.9 Comparing Technologies

Technology	Substitution	Isoquant Shape	Example
Perfect substitutes	High	Linear	Labor vs machines
Cobb–Douglas	Moderate	Smooth, convex	Capital and labor
Fixed proportions	None	L-shaped	Assembly lines

Table 11.1: Comparison of production technologies

## 11.10 Examples

### Example 1: Marginal Product

Suppose:

$$f(x_1, x_2) = x_1^{1/2} x_2^{1/2}.$$

Then:

$$MP_1 = \frac{1}{2} x_1^{-1/2} x_2^{1/2}, \quad MP_2 = \frac{1}{2} x_1^{1/2} x_2^{-1/2}.$$

As  $x_1$  increases,  $MP_1$  decreases, illustrating diminishing marginal productivity.

### Example 2: MRTS

Using the same function:

$$MRTS_{12} = \frac{MP_1}{MP_2} = \frac{x_2}{x_1}.$$

The MRTS falls as  $x_1$  increases relative to  $x_2$ , confirming convex isoquants.

## Chapter 12: Profit Maximization

### 12.1 The Firm's Problem

The firm's objective is to choose inputs and output to maximize profit. Profit is defined as total revenue minus total cost:

$$\pi = py - w_1x_1 - w_2x_2,$$

where:

- $p$  is the output price,
- $y$  is output,
- $w_1, w_2$  are input prices,
- $x_1, x_2$  are inputs,
- $y = f(x_1, x_2)$  is the production function.

The firm's problem can be written as:

$$\max_{x_1, x_2} \pi = pf(x_1, x_2) - w_1x_1 - w_2x_2.$$

This problem combines technology (production) and market prices.

### 12.2 Two-Step Logic of Profit Maximization

Profit maximization can be broken into two steps:

1. For any given output level  $y$ , choose inputs to minimize cost.
2. Choose the output level  $y$  that maximizes profits.

This chapter focuses on the first step: choosing inputs optimally.

### 12.3 Profit Maximization with One Variable Input

Suppose input  $x_2$  is fixed in the short run. The production function becomes:

$$y = f(x_1, \bar{x}_2).$$

Profit is:

$$\pi(x_1) = pf(x_1, \bar{x}_2) - w_1x_1 - w_2\bar{x}_2.$$

The first-order condition is:

$$p \cdot MP_1 = w_1,$$

where  $MP_1 = \frac{\partial f}{\partial x_1}$ .

**Interpretation:** Hire labor until the value of its marginal product equals its wage.

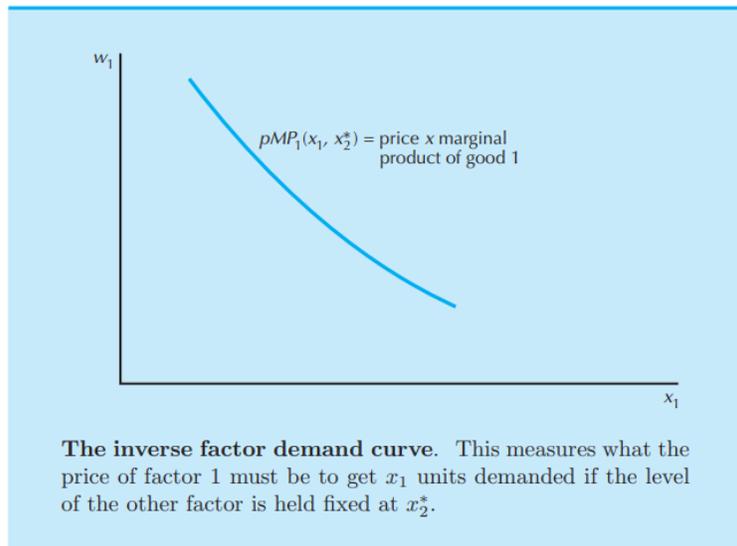


Figure 12.1: Inverse factor demand curve:  $p \cdot MP_1(x_1)$

The curve  $p \cdot MP_1$  is the firm's inverse demand for input  $x_1$ .

### 12.4 Comparative Statics of Input Demand

Changes in prices affect the firm's optimal input choice:

- Higher  $w_1$  reduces input demand.
- Higher output price  $p$  increases input demand.

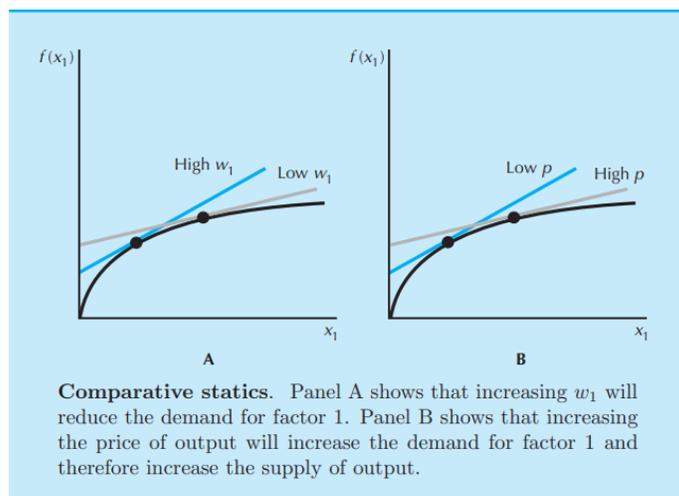


Figure 12.2: Comparative statics of factor demand

**Intuition:** A higher wage makes the input more expensive, so the firm substitutes away. A higher output price raises the value of production, making inputs more valuable.

## 12.5 Profit Maximization with Isoprofit Lines

Rewrite profit as:

$$y = \frac{\pi}{p} + \frac{w_1}{p}x_1 + \frac{w_2}{p}x_2.$$

This is a straight line in  $(x_1, y)$  space called an **isoprofit line**.

- Slope:  $\frac{w_1}{p}$
- Higher isoprofit lines mean higher profits

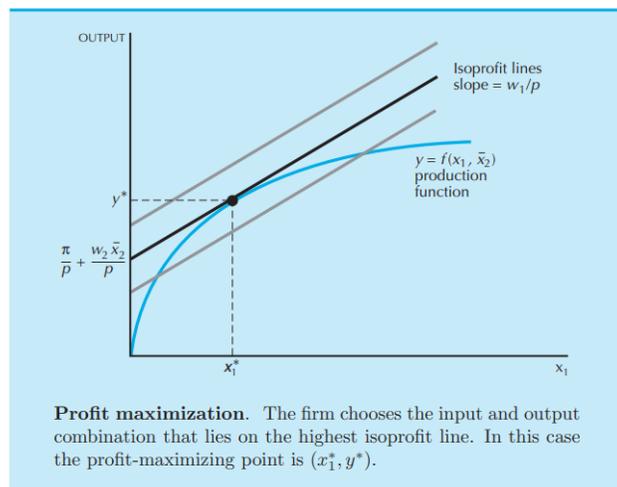


Figure 12.3: Profit maximization using isoprofit lines

**Optimal choice:** The firm chooses the point where the highest isoprofit line is tangent to the production function.

## 12.6 First-Order Condition (Interior Solution)

At the tangency:

$$\frac{MP_1}{MP_2} = \frac{w_1}{w_2}.$$

Equivalently:

$$p \cdot MP_1 = w_1 \quad \text{and} \quad p \cdot MP_2 = w_2.$$

**Economic meaning:** Each input is hired until its value marginal product equals its price.

## 12.7 Relationship to Cost Minimization

Profit maximization implies cost minimization.

If a firm does not minimize cost for a given output level, it could increase profit by rearranging inputs.

Thus:

Profit maximization  $\Rightarrow$  Cost minimization.

## 12.8 Short Run vs Long Run

**Short run:**

- At least one input is fixed
- Adjustment happens only through variable inputs

**Long run:**

- All inputs are variable
- Firm chooses the cost-minimizing input mix

Long-run profits are always at least as high as short-run profits.

## 12.9 Example 1: Cobb–Douglas Technology

Let:

$$f(x_1, x_2) = x_1^{1/2} x_2^{1/2}, \quad p = 10, \quad w_1 = 2, \quad w_2 = 8.$$

First-order conditions:

$$10 \cdot \frac{1}{2} x_1^{-1/2} x_2^{1/2} = 2,$$

$$10 \cdot \frac{1}{2} x_1^{1/2} x_2^{-1/2} = 8.$$

Taking the ratio:

$$\frac{x_2}{x_1} = \frac{w_1}{w_2} = \frac{2}{8} = \frac{1}{4}.$$

Thus:

$$x_2 = \frac{1}{4} x_1.$$

This gives the optimal input mix.

## 12.10 Example 2: Effect of a Wage Increase

Suppose  $w_1$  increases.

- The condition  $p \cdot MP_1 = w_1$  now requires lower  $MP_1$
- Lower  $MP_1$  implies less  $x_1$

**Conclusion:** The demand for labor slopes downward.

## In Short

Profit maximization means choosing inputs so that:

$$p \cdot MP_i = w_i \quad \text{for each input } i.$$

The firm balances technological trade-offs with market prices to achieve the highest possible profit.

# Chapter 13: Cost Minimization

## 13.1 Purpose and Roadmap

A profit-maximizing firm can be analyzed in two steps:

1. **Cost minimization (this chapter):** For any target output  $y$ , choose inputs to produce  $y$  at the lowest possible cost. This delivers the *cost function*  $c(\mathbf{w}, y)$  and the *conditional factor demands*.
2. **Profit maximization (previous chapter):** Choose  $y$  to maximize  $\pi(p, \mathbf{w}) = py - c(\mathbf{w}, y)$  using the cost function from step 1.

### Core Intuition

Technology gives *isoquants* (all input bundles yielding the same output). Prices give *isocost lines* (all bundles with the same total cost). The least-cost way to produce a given  $y$  is where the isoquant for  $y$  is tangent to the lowest possible isocost. At that point, the *technical trade-off* equals the *market trade-off*.

## 13.2 The Firm's Cost-Minimization Problem

Let  $\mathbf{x} = (x_1, x_2)$  be inputs with prices  $\mathbf{w} = (w_1, w_2)$  and production function  $y = f(x_1, x_2)$ . The problem is

$$\min_{x_1, x_2} w_1x_1 + w_2x_2 \quad \text{s.t.} \quad f(x_1, x_2) = y.$$

The minimized value defines the *cost function*

$$c(\mathbf{w}, y) \equiv \min_{\{x_1, x_2: f(x_1, x_2)=y\}} \{w_1x_1 + w_2x_2\},$$

and the optimizer  $(x_1^*(\mathbf{w}, y), x_2^*(\mathbf{w}, y))$  are the *conditional (derived) factor demands*.

### 13.2.1 Isoquants and Isocost Lines

- **Isoquant for  $y$ :**  $\{(x_1, x_2) : f(x_1, x_2) = y\}$ . Downward sloping and typically convex (diminishing TRS).
- **Isocost for total cost  $C$ :**  $w_1x_1 + w_2x_2 = C \Rightarrow x_2 = \frac{C}{w_2} - \frac{w_1}{w_2}x_1$ , slope  $-\frac{w_1}{w_2}$ .

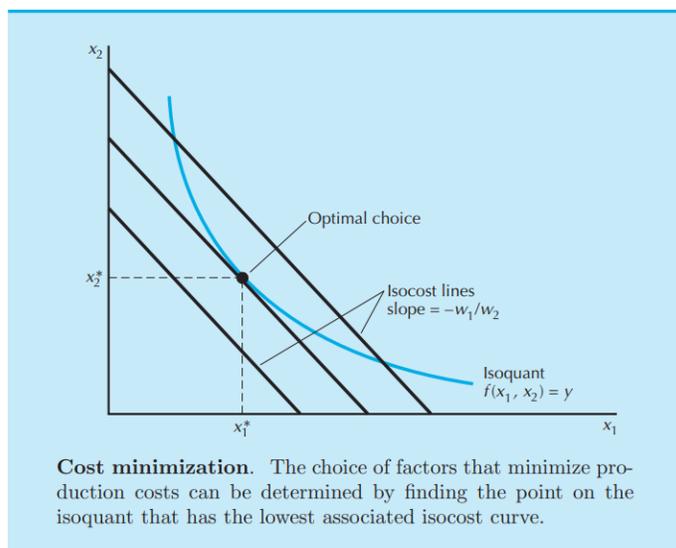


Figure 1: Tangency of isoquant and isocost at the least-cost bundle.

**Tangency (first-order) condition.** At an interior solution, the *technical rate of substitution* equals the input-price ratio:

$$\text{TRS}(x_1^*, x_2^*) = -\frac{MP_1}{MP_2} = -\frac{w_1}{w_2}.$$

*Economic meaning:* at the optimum, the marginal cost of saving one unit of  $x_2$  by using more  $x_1$  (*technology*) matches the market rate at which  $x_1$  can substitute for  $x_2$  (*prices*). Equivalently,

$$\frac{MP_1}{w_1} = \frac{MP_2}{w_2}$$

so each dollar of spending produces the same marginal output (“equal bang per buck”).

### 13.2.2 Algebraic Intuition for the Rule

Holding output fixed ( $\Delta y = 0$ ) implies

$$MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0.$$

At the cost minimum, no feasible change can lower cost:

$$w_1 \Delta x_1 + w_2 \Delta x_2 = 0.$$

Solving gives

$$\frac{MP_1}{MP_2} = \frac{w_1}{w_2},$$

the same tangency condition.

### 13.2.3 Conditional Factor Demands and the Cost Function

For  $i \in \{1, 2\}$ ,

$$x_i^* = x_i(\mathbf{w}, y), \quad c(\mathbf{w}, y) = w_1 x_1(\mathbf{w}, y) + w_2 x_2(\mathbf{w}, y).$$

*Comparative statics:* when  $w_i$  rises (holding  $y$  fixed),  $x_i^*$  falls (own-price effect  $< 0$ ). When  $y$  rises, both  $x_1^*, x_2^*$  weakly increase.

### 13.3 Cost Functions

**(i) Perfect Complements:**  $f(x_1, x_2) = \min\{x_1, x_2\}$

To produce  $y$  you must have  $x_1 = x_2 = y$ . Hence

$$x_1^* = y, \quad x_2^* = y, \quad c(\mathbf{w}, y) = (w_1 + w_2) y.$$

*Geometry:* L-shaped isoquants; optimum at the kink.

**(ii) Perfect Substitutes:**  $f(x_1, x_2) = x_1 + x_2$

Any mix with  $x_1 + x_2 = y$  works. Use only the cheaper input:

$$c(\mathbf{w}, y) = \min\{w_1, w_2\} y.$$

If  $w_1 < w_2$ , then  $x_1^* = y, x_2^* = 0$ ; reverse if  $w_2 < w_1$ . Corner solutions are typical.

**(iii) Cobb–Douglas:**  $f(x_1, x_2) = Ax_1^a x_2^b$ , with  $a, b > 0$

Tangency gives

$$\frac{a}{b} \frac{x_2}{x_1} = \frac{w_1}{w_2} \quad \Rightarrow \quad x_2 = \frac{bw_1}{aw_2} x_1.$$

Impose  $Ax_1^a x_2^b = y$  and solve for the cost-minimizing bundle:

$$x_1^*(\mathbf{w}, y) = \frac{a}{a+b} \frac{y}{A^{1/(a+b)}} \left(\frac{w_2}{w_1}\right)^{\frac{b}{a+b}}, \quad x_2^*(\mathbf{w}, y) = \frac{b}{a+b} \frac{y}{A^{1/(a+b)}} \left(\frac{w_1}{w_2}\right)^{\frac{a}{a+b}}.$$

The (unscaled) Cobb–Douglas cost function is

$$c(\mathbf{w}, y) = K(a, b, A) w_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}},$$

where  $K(a, b, A) = a^{-a} b^{-b} A^{-1/(a+b)}$ .

Technology	Production Function	Cost Function $c(w_1, w_2, y)$	Comment
Perfect Complements	$f(x_1, x_2) = \min\{x_1, x_2\}$	$(w_1 + w_2) y$	Inputs used in 1:1 ratio
Perfect Substitutes	$f(x_1, x_2) = x_1 + x_2$	$\min\{w_1, w_2\} y$	Use only cheaper input
Cobb–Douglas	$f(x_1, x_2) = A x_1^a x_2^b$	$K w_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$	Smooth, diminishing TRS

### 13.4 Revealed Cost Minimization (WACM)

Suppose prices change from  $\mathbf{w}^t$  to  $\mathbf{w}^s$  while output stays fixed at  $y$ , and the observed choices are  $\mathbf{x}^t$  and  $\mathbf{x}^s$ . If both are cost-minimizing, then

$$(\mathbf{w}^t)^\top \mathbf{x}^t \leq (\mathbf{w}^t)^\top \mathbf{x}^s \quad \text{and} \quad (\mathbf{w}^s)^\top \mathbf{x}^s \leq (\mathbf{w}^s)^\top \mathbf{x}^t.$$

Adding yields the useful implication

$$(\Delta w_1)(\Delta x_1) + (\Delta w_2)(\Delta x_2) \leq 0,$$

so cost-minimizing input choices move *opposite* their own price changes.

### 13.4.1 Short Run vs. Long Run

**Short run (some inputs fixed).** If  $x_2$  is fixed at  $\bar{x}_2$ ,

$$c_s(y; \bar{x}_2) = \min_{x_1} \{w_1 x_1 + w_2 \bar{x}_2\} \quad \text{s.t.} \quad f(x_1, \bar{x}_2) = y,$$

with short-run demand  $x_1^s(w_1, w_2, \bar{x}_2, y)$ .

**Long run (all inputs variable).**

$$c(\mathbf{w}, y) = \min_{x_1, x_2} \{w_1 x_1 + w_2 x_2\} \quad \text{s.t.} \quad f(x_1, x_2) = y.$$

Relationship: the long-run cost equals the short-run cost evaluated at the long-run optimal fixed factor,

$$c(\mathbf{w}, y) = c_s(y; x_2^*(\mathbf{w}, y)),$$

so  $c(\mathbf{w}, y) \leq c_s(y; \bar{x}_2)$  for any  $\bar{x}_2$  (flexibility cannot raise the minimum cost).

### 13.4.2 Returns to Scale and the Shape of Costs

If  $f$  has increasing returns to scale (IRS), then as  $y$  rises the average cost  $AC(y) = c(\mathbf{w}, y)/y$  tends to fall (economies of scale). Under CRS,  $AC$  is flat; under DRS,  $AC$  rises. Hence technology's returns to scale map directly into cost behavior.

### 13.4.3 Sunk Costs vs. Flow (Operating) Costs

*Sunk costs* are expenditures that cannot be recovered; they should not affect forward-looking choices. Evaluate production using *flow costs* (current operating costs). This keeps the cost-minimization problem economically meaningful.

### 13.4.4 Key Conditions and Relationships

Concept	Equation / Condition	Meaning
Tangency (interior)	$-\frac{MP_1}{MP_2} = -\frac{w_1}{w_2}$	Equalize "bang per buck."
Cost function	$c(\mathbf{w}, y)$	Minimum cost for output $y$ .
Conditional demands	$x_i(\mathbf{w}, y)$	Least-cost input quantities.
Short-run cost	$c_s(y; \bar{x}_2)$	Some inputs fixed.
WACM	$(\Delta w_1)(\Delta x_1) + (\Delta w_2)(\Delta x_2) \leq 0$	Opposite price–quantity movement.

## 13.5 Some Worked Examples

### Example 1: Perfect Substitutes

A firm has  $f(x_1, x_2) = x_1 + x_2$ , prices  $(w_1, w_2) = (4, 6)$ , and target  $y = 100$ .

- Cheapest input is  $x_1$  ( $4 < 6$ ). Use only  $x_1$ :

$$x_1^* = 100, \quad x_2^* = 0, \quad c(\mathbf{w}, y) = 4 \times 100 = 400.$$

- If instead  $(w_1, w_2) = (8, 5)$ , switch:  $x_1^* = 0, x_2^* = 100, c = 500$ .

*Intuition:* linear isoquants  $\Rightarrow$  corner solutions; pick the cheaper input.

**Example 2: Perfect Complements**

Suppose  $f(x_1, x_2) = \min\{x_1, x_2\}$  with  $(w_1, w_2) = (2, 3)$  and  $y = 40$ .

$$x_1^* = x_2^* = 40, \quad c(\mathbf{w}, y) = (2 + 3) \times 40 = 200.$$

*Intuition:* fixed proportions; the isoquant kink pins you at  $x_1 = x_2$ .

**Example 3: Cobb–Douglas: conditional demands and cost**

Let  $f(x_1, x_2) = x_1^{0.5}x_2^{0.5}$ ,  $(w_1, w_2) = (4, 1)$ , and  $y = 10$ .

1. Tangency:  $\frac{MP_1}{MP_2} = \frac{x_2}{x_1} = \frac{w_1}{w_2} = 4 \Rightarrow x_2 = 4x_1$ .
2. Feasibility:  $x_1^{0.5}(4x_1)^{0.5} = 10 \Rightarrow \sqrt{4}x_1 = 10 \Rightarrow x_1^* = 5, x_2^* = 20$ .
3. Minimum cost:  $c = 4 \cdot 5 + 1 \cdot 20 = 40$ .

*Intuition:* spend more on the input with higher  $MP$  per dollar until equalized.

## Chapter 14: Cost Curves

### Big Picture

This chapter translates technology and input prices into *cost functions* and the familiar curves firms use for decisions in the short run and long run. We distinguish total, average, and marginal costs; link curve shapes to diminishing returns; and show how long-run costs envelope short-run costs when all factors are variable.

#### 14.0.1 Cost Building Blocks

- **Total cost (TC):**  $C(y) = F + VC(y)$ , where  $F$  is fixed (does not vary with  $y$ ) and  $VC(y)$  is variable.
- **Average cost (AC):**  $AC(y) = \frac{C(y)}{y} = \frac{F}{y} + \frac{VC(y)}{y} = AFC(y) + AVC(y)$ .
- **Average fixed cost (AFC):**  $AFC(y) = \frac{F}{y}$  (strictly decreasing in  $y$ ).
- **Average variable cost (AVC):**  $AVC(y) = \frac{VC(y)}{y}$ .
- **Marginal cost (MC):**  $MC(y) = \frac{dC(y)}{dy} = \frac{dVC(y)}{dy}$  (fixed cost drops out).

#### Key Identities

$$AC = AFC + AVC, \quad MC = \frac{dC}{dy}, \quad \text{and for any } y_1 < y_2: VC(y_2) - VC(y_1) = \int_{y_1}^{y_2} MC(q) dq.$$

#### 14.0.2 Short Run: Why the Curves Look the Way They Do

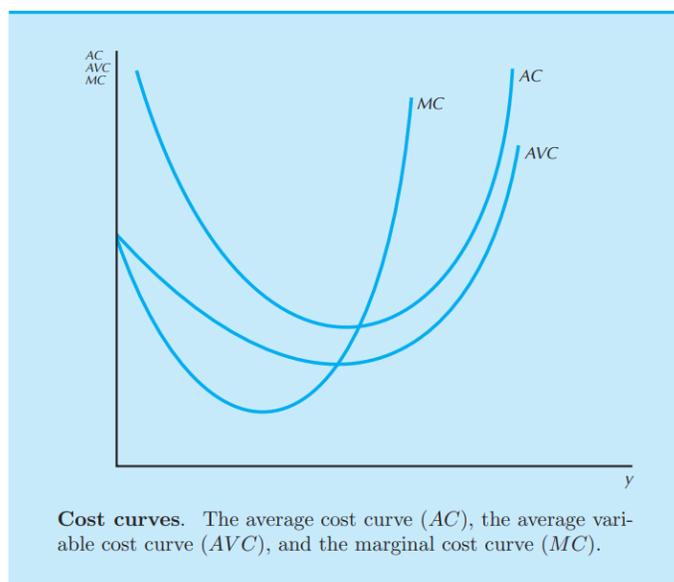
In the short run, at least one input (e.g., capital  $K$ ) is fixed. If labor  $L$  is variable at wage  $w$ ,

$$SMC(y) = \frac{w}{MPL}, \quad AVC(y) = \frac{w}{APL},$$

so diminishing marginal product ( $MPL \downarrow$ ) pushes  $SMC \uparrow$  and eventually makes  $AVC \uparrow$ . This yields the familiar U-shapes of  $AVC$  and  $AC$ .

#### Intersection Rules (must know).

- $MC$  cuts  $AVC$  at  $AVC$ 's minimum.
- $MC$  cuts  $AC$  at  $AC$ 's minimum.
- When  $MC < AC$  (or  $< AVC$ ), the average falls; when  $MC > AC$ , the average rises.

Figure 14.1: Cost curves:  $AC$ ,  $AVC$ , and  $MC$ .

**Area Relationship.** The *area under* the  $MC$  curve from 0 to  $y$  equals  $VC(y)$ . Graphically, this accumulates marginal spending on each extra unit.

### 14.0.3 Long Run: Envelopes of Short-Run Costs

When all inputs are adjustable, the firm can choose the cost-minimizing capital for each  $y$ . The long-run average cost  $LAC(y) = \frac{C(y)}{y}$  is the *lower envelope* of all feasible short-run average cost curves  $SAC(y; K)$ . For any output  $y^*$ , one short-run plant choice  $K^*$  makes  $SAC(\cdot; K^*)$  *tangent* to  $LAC$ .

**Economies vs. Diseconomies of Scale.** Define the *cost elasticity*  $E_c(y) \equiv \frac{MC(y)}{AC(y)}$ .

$$\begin{cases} E_c(y) < 1 & \Rightarrow \text{economies of scale } (LAC \downarrow) \\ E_c(y) = 1 & \Rightarrow \text{constant returns at the cost level } (LAC \text{ flat}) \\ E_c(y) > 1 & \Rightarrow \text{diseconomies of scale } (LAC \uparrow) \end{cases}$$

### 14.0.4 Shutdown and Operating Decisions (Short Run)

- **Shutdown price:**  $p < \min AVC \Rightarrow$  produce  $y = 0$  (avoid variable losses).
- If  $p \geq \min AVC$ : produce where  $p = SMC(y)$  on the upward-sloping part, even if  $p < AC$  (you cover variable costs and part of fixed costs).

### 14.0.5 Worked Examples

**Example 1 (algebraic curves from a cost function).** Let  $C(y) = 100 + 4y + 0.5y^2$ .

$$FC = 100, \quad VC(y) = 4y + 0.5y^2, \quad AFC = \frac{100}{y}, \quad AVC = 4 + 0.5y,$$

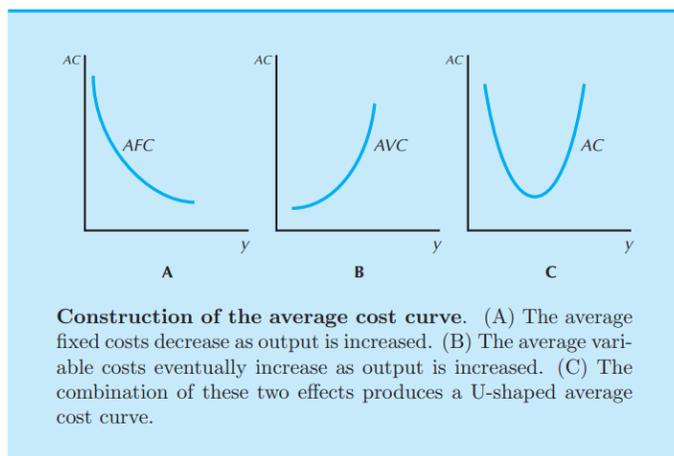


Figure 14.2: Why  $AC$  is U-shaped:  $AFC$  falls as output rises;  $AVC$  eventually rises; adding them gives U-shaped  $AC$ .

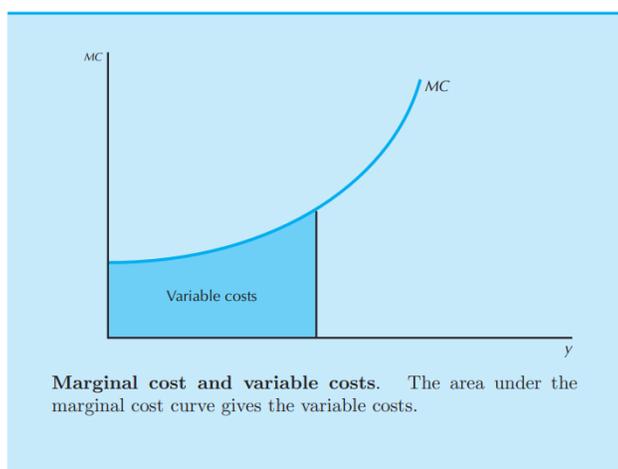


Figure 14.3: Variable cost as the area under  $MC$ .

$$AC = \frac{100}{y} + 4 + 0.5y, \quad MC = 4 + y.$$

*Minima:*

$$\text{Set } \frac{dAC}{dy} = -\frac{100}{y^2} + 0.5 = 0 \Rightarrow y_{AC}^{\min} = \sqrt{\frac{200}{1}} = \sqrt{200} \approx 14.14.$$

At this  $y$ ,  $AC = MC = 4 + y \approx 18.14$ . For  $AVC$ ,  $\frac{dAVC}{dy} = 0.5 = 0$  has no interior solution; here  $AVC$  is linear and increasing, so  $\min AVC$  occurs at the smallest feasible  $y$  (common in simple quadratics with linear  $AVC$ ). In richer SR settings with fixed  $K$ ,  $AVC$  is U-shaped and  $MC$  crosses at its bottom.

**Example 2 (shutdown threshold).** Suppose short-run cost is  $C(y) = 80 + 2y + y^2$ . Then

$$AVC = \frac{2y + y^2}{y} = 2 + y, \quad MC = 2 + 2y.$$

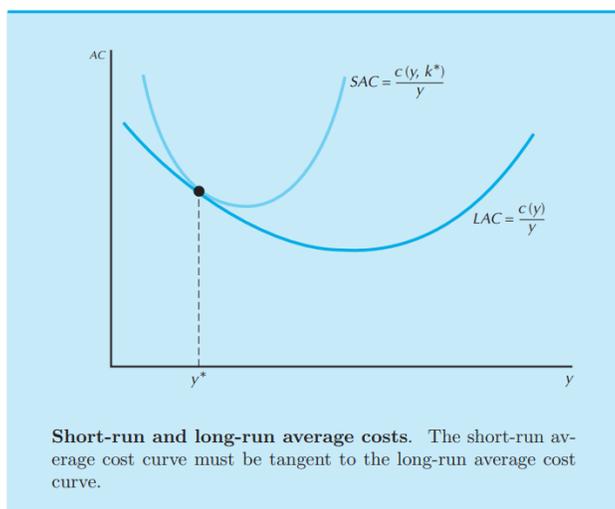


Figure 14.4: Long-run envelope:  $SAC$  is tangent to  $LAC$  at the plant's efficient scale  $y^*$ .

$\min AVC$  occurs at the smallest feasible  $y$  (again linear), giving a shutdown price at the lowest attainable  $AVC$ . If the technology implied  $AVC$  U-shaped, we'd set  $SMC = AVC$  to find  $\min AVC$  and compare  $p$  to that value.

**Example 3 (link to production).** Short run with wage  $w = 20$  and production  $y = f(L, K) = K^{1/2}L^{1/2}$  where  $K$  is fixed at  $K = 25$ . Then

$$y = 5\sqrt{L} \Rightarrow L = \left(\frac{y}{5}\right)^2, \quad VC = wL = 20\left(\frac{y}{5}\right)^2 = \frac{4}{5}y^2.$$

Hence

$$AVC = \frac{VC}{y} = \frac{4}{5}y, \quad SMC = \frac{dVC}{dy} = \frac{8}{5}y, \quad AC = \frac{F}{y} + \frac{4}{5}y.$$

All shapes follow from  $MPL \downarrow$  as  $L$  rises with fixed  $K$ .

### 14.0.6 What Moves the Curves

- **Input prices:** an increase in an input price (e.g.,  $w$ ) shifts  $VC$ ,  $AVC$ ,  $MC$  up proportionally to that input's cost share;  $FC$  and  $AFC$  move with the price of fixed inputs.
- **Technology:** higher productivity (e.g., larger  $MPL$ ) shifts  $MC$ ,  $AVC$ ,  $AC$  down; process innovations flatten curves.
- **Plant size (SR):** choosing a different  $K$  traces a different  $SAC$ ; the collection of all  $SAC$  envelopes to  $LAC$ .

### 14.0.7 Compact Review

- $AC = AFC + AVC$ ;  $MC$  ignores fixed cost.
- Area under  $MC$  is  $VC$ .
- $MC$  intersects  $AVC$  and  $AC$  at their minima.
- Long run is the envelope of short run:  $LAC = \min_K SAC(\cdot; K)$ .
- Economies of scale when  $MC < AC$  (cost elasticity  $< 1$ ).

### **Acknowledgment and Source Notice**

These lecture notes were created for personal academic study and educational use in ECO 302: Intermediate Microeconomics. They are based on concepts, figures, and examples presented in *Intermediate Microeconomics: A Modern Approach, 8th Edition* by Hal R. Varian (University of California, Berkeley).

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